Mathematics Teacher

DEVOTED TO THE INTERESTS OF MATHEMATICS
IN JUNIOR AND SENIOR HIGH SCHOOLS

VOLUME XX JANUARY, 1927 NUMBER 1

Published by the

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

NEW YORK YONKERS, N. Y.

Entered as second-class matter, November 10, 1921, at the Post Office at Yonkers, N. Y., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized November 17, 1921.

The Mathematics Teacher

Official Journal of the National Council of Teachers of Mathematics

This organization has for its object the advancement of mathematics teaching in junior and senior high schools. All persons interested in mathematics and mathematics teaching are eligible to membership. All members receive the official journal of the National Council—The Mathematics Teacher—which appears monthly, except June, July, August and September.

Correspondence relating to editorial matters, subscriptions, advertisements, and other business matters should be addressed to

John R. Clark, Editor-in-Chief
The Lincoln School of Teachers College
425 West 123rd Street, New York City

[illegible] $2.00 per year (eight numbers)
[illegible] Canadian postage, 25 cents per year.
[illegible] cents should be added for exchange.
[illegible] 40 cents.

General
Education
Waln

THE MATHEMATICS TEACHER

VOLUME XX JANUARY, 1927 NUMBER 1

AN OPEN LETTER

Dec. 26, 1926.

Professor Franklin Bobbitt,
University of Chicago,
Chicago, Illinois.

My dear Professor Bobbitt:

Some months ago I took home your book, HOW TO MAKE A CURRICULUM. I opened it at the chapter on mathematics and read with interest until I came to "Guiding Principles and Assumptions." After reading these I found myself in vigorous opposition to you on some points. Hence this letter. And since your book has and, in most particulars, deserves both popularity and authority, I am making this an open letter and sending it to a mathematical journal.

The sections to which I most strenuously object are the following:

"21. Outside of certain specialized technical occupations, trigonometry is not used. It does not provide centers for systems of ideas valuable in one's intellectual or humanistic life. The vision and inspiration presented by the right triangle is scarcely worth mentioning."

"31. As fields of intellectual play, neither algebra, demonstrative geometry, nor trigonometry provide centers of growing systems of ideas of any considerable humanistic value. As apperception centers they do not greatly function."

"33. Mathematics is a high intellectual realm which makes a fit field of intellectual play experience on a high level for the rarer, finer and more capable minds. Those who can scale these austere heights with zest and pleasure, mostly self-directed, and with such speed that time needed for other things is not disproportionately consumed, can doubtless profit from the intellectual experience. Whether the profit be small or large is unknown."

"35. In considering mathematics for developing intellectual power and endurance, it should be borne in mind that there are numerous other heights to be scaled which are as complex and difficult as mathematics; and from which, when once scaled, the outlook is more profitable."

In opposition to these assumptions of yours I maintain that as apperception centers mathematics functions as greatly as any department of human knowledge or experience. And in support of my contention, I give the following historical examples. I pass by Pythagoras, whose name is inseperably connected with the right triangle, and whose whole philosophy is based on the idea that mathematical relations are at the foundation of all

things. Of the Greek philosophers I will consider only the greatest, Plato. It was with reason that Plato warned from the Academy those who did not know geometry. The "idea" is the essence of Plato's philosophy and had its birth in Geometry, for in the philosophy of Plato the idea, horse, bears the same relation to physical horses that the triangle of geometry bears to three-sided sheets of metal or paper. On this point let me quote from Weber's History of Philosophy (page 80 of Thilly's translation):

"Geometry made a particularly deep impression on him; the geometrical method served as a model for his metaphysica. Indeed he even borrowed his philosophical vocabulary from that science."

I shall give one more example from the same field. The philosophy of Immanuel Kant had its origin in mathematics. In his "Prolegomena" Kant tells how he came to work out and write his "Critique of Pure Reason." We will let Kant speak for himself. I quote here and there from the translation of the "Prolegomena" by Paul Carus.

Page 7. "I openly confess the suggestion of David Hume was the very thing which many years ago interrupted my dogmatic slumber and gave my investigations in the field of speculative philosophy quite a new direction."

Page 21. "Hume, being prompted (a task worthy of a philosopher) to cast his eye over the whole field of a priori cognitious in which human understanding claims such mighty possessions, heedlessly severed from it a whole, and indeed its most valuable, province, viz., pure mathematics.

In this, however, he was greatly mistaken, and the mistake had a decidedly injurious effect upon his whole conception. But for this, he would have extended his question concerning our synthetical judgments far beyond the metaphysical concept of Casuality, and included in it the possibility of mathematics a priori also. Thus that astute man would have been led into considerations whch must needs be similar to those that now occupy us, but which would have gained inestimably by his inimitably elegant style."

Out of mathematical considerations, then, grew the philosophy of Immanuel Kant. And the first chapter of the "First Part of the Transcendental Problem" in the Prolegomena has the heading, "How is Pure Mathematics Possible?"

In these two cases at least mathematics seems to have furnished rather productive apperceptive centers. Is it not reasonable to allow some considerable possibilities in the same direction to lesser minds? Ought you not at least to qualitfy your statements?

It may be, however, that you put philosophy in the same field as mathematics, i.e., the "field of intellectual play experience."

And even in reference to mathematics, can it be that I understand you aright? Do you put the higher mathematics in the same category as bridge and chess? Do you really look upon abstract mathematics as a very dignified game of solitaire, that those who like that kind of game may play, provided of course that "time needed for other things is not disproportionately consumed"? And as you think of the "numerous other heights to be scaled which are as complex and difficult as mathematics; and from which, when once scaled, the outlook is more profitable," do you, as I imagine, sadly shake your head as you write, "Whether the profit be small or large is unknown?"

Here also I join issue with you. Let me give you an example or two of the part these games of solitaire have played in the history of civilization. From the days of the ancient Greeks through more than a decade of centuries, mathematicians played with conic sections. During all of this time they were no more useful than a game of chess. By Kepler's time this game of solitaire had been wonderfully developed. Fortunately, Kepler was an expert at it. Fortunately, also, to him it was more than a game, and poet that he was, he united it with the observations left by Tycho Brahe and gave astronomy Kepler's three laws.

Some time also in the unrecorded middle ages a man tried to work a problem which led to the subtraction of a larger number from a smaller. Instead of simply regarding it as impossible, he called it a negative quantity. He and others played with these impossible things, these negative quantities. Gradually they came to have a meaning; and today modern mechanics and modern physics would be helpless if deprived of negative quantities.

Later still, one more example must suffice, mathematicians found another plaything, the square root of a negative quantity. Even to this day they call it an "imaginary." Finally, however, it was given an interpretation and now scientists are using it right along.

But what are the heights "from which, when once scaled, the outlook is more profitable?" Can you suggest such a height for Plato, or Kant, or Kepler? Can you really doubt whether, in their cases, the profit was small or large?

Let me suggest that not music, nor art, nor poetry, nor literature, nor science, nor mathematics is to be given any precedence in the contribution they are capable of making to our humanistic life. The fact that the right triangle means little to you does not prevent its opening up whole vistas to me. I see the right triangle dividing up the western part of the United States by section lines, and pointing out the location in every deed of land. I see the right triangle guiding commerce over the seven seas. I see the right triangle measuring and weighing the earth, the sun, and the planets. I see the right triangle conquering distances almost unthinkable and measuring the stars by light that left them years, and even centuries ago. When I turn on the radio set, I see the right triangle helping Maxwell develop the electro-magnet theory of light. A week ago I attended a concert in which there were obligatos by the violin and the clarinet. Any one can distinguish the difference in tone between a violin and a clarinet. But if you ask me in just what that difference consists, I think of Helmholtz's analysis of sound and sine curves, the children of the right triangle, which may be used to represent anything that is periodic in nature. And this idea of periodicity suggests rhythm and poetry; for the cadence as one reads poetry is one of the countless forms of wave motion. But why go into further details? The connotations of the right triangle are to be found everywhere. For those who have eyes to see, the figure of the right triangle may be found among philosophers and poets, among classical musicians and ragtimers, among Greek architects and cubist artists.

In future editions of your book, will it not be well to eliminate or at least seriously modify some of your "Guiding Principles and Assumptions?"

I will even be so rash as to suggest as a guiding principle or assumption that from every field of human knowledge there rise heights from which the mountain tops at least in every other field may be seen, provided of course the climber will occasionally stop, take his eyes off the ground where he is next to plant his feet and look around.

Yours very truly,

Hiram B. Loomis,

Principal, Hyde Park High School, Chicago, Ill.

FADS AND PLANE GEOMETRY

H. D. MERRELL
High School, Evanston, Ill.

I

The other day Mrs. Merrell discovered our six-year-old daughter wrapping her toothbrush in a piece of paper.

"Kate-Alice, what are you doing with your toothbrush?"

"Teacher said we must bring our toothbrushes to school."

"You will do no such thing. School is no place for toothbrushes."

I think she was right. So far as our family is concerned, the toothbrush responsibility is a home responsibility. Also every intelligent member of the school mothers' club knows that the toothbrush is not the first defense against bad teeth. It was invented by the devil to reconcile his victims to their dietary sins. A campaign against all-day suckers would be more to the point. But such a campaign would be unpopular; and also distasteful to the amateur uplifter.

Another feature of my daughter's program is a French lesson, twice a week I believe, in spite of the well-known fact that half her class will not be able to add two fractions when they are sixteen years old.

Our public schools are as subject to fads as a woman's wardrobe and one reason is that in many communities the good women control the schools. At least it is an open question whether the superintendent is bossing the job or serving as office boy. A woman remarked in my hearing once: "Mr. Blank thinks he is running that school, but he is not. *We* are running it." A grade teacher said to her friend: "We don't pay much attention to the superintendent. Superintendents come and go, but parents stay on forever." The sad fact is many superintendents are not big enough for the job, a state of affairs for which there is a perfectly good economic explanation.

The psychologists are responsible for many fads. Of course, we must have these experimenters, these forward-looking people, but they have the very unfortunate habit of serving new concoctions to us before they are thoroughly cooked and tested as to food values. Not so long ago one of these eminent gentlemen

announced that his experiments did not indicate any transfer of discipline. The nation-wide conflagration that ensued would have caused O'Leary's cow to blush with shame. At least one great state abolished all required mathematics in its high schools, and individual schools everywhere performed major or minor operations. After it was too late, the eminent psychologists performed more experiments and decided there must be some transfer of discipline,—a fact that had been fairly obvious to the man in the street all the while. Any school man of twenty years experience can recall a dozen intriguing theories that have sprung up, had their tragic voyage, and died out like a prairie fire, leaving more or less desolation in their wake. One healthy mother's club, one common or garden variety of superintendent, one half-baked pedagogical theory, five hundred students whose hands have not been soiled in manual labor and whose brains have not been guilty of thought,—these are the standard ingredients of educational moonshine.

Twenty-five years ago Robert J. Aley lectured on *Educational Fads*. He said that most of these pretty devices were foredoomed to failure because they were merely ingenious attempts to side step one of Nature's age-old laws. The biologist calls this law *The Survival* of the *Fittest*. Christ stated it: "To him that hath it shall be given, etc." Common folks say: "You must work or starve." Every philosopher, every scientist, every honest man since the dawn of civilization has insisted that you cannot get something for nothing. Yet it is exactly that gambler's hope that prompts many of these new movements.

The outstanding fad of recent years, I suppose, was the craze for manual training and so-called vocational courses. It fired the popular imagination until even the strongest superintendents were put to it to control their communities. Now the pendulum has started its downward swing.

Germany and France from whom we borrowed the vocational idea, are dropping it, and sentiment in this country is following suit. The editor of the *Chicago Tribune* said recently: "The ideal of our public schools should be an educated people, not a trained populace." President Lewis of the Coyne Electrical School should be a rabid vocational fan. But listen:

"Education as a whole, particularly the first twelve grades, is education of the mind. During these grades far less thought is given to what profession a student is going to enter than to the immediate task in hand, that of giving the mind training and exercise." All of you can point to schools that have abandoned, or would like to abandon, some of their vocational courses.

The latest fad, I believe, is the *Intelligence Test*. I have no opinion regarding the ultimate worthwhileness of these tests. But I know that they are in the experimental stage. The College Entrance Board offered them last year, for the first time. And why? Because it seemed desirable to have an authoritative and disinterested background for further investigation into their merits. There are proper people and proper places for making these investigations. The rank and file of teachers did not buy automobiles in the early days of automobile development. They knew they could not afford to. They waited until the industry was stabilized. The public schools, the common people's schools, should take the same attitude toward these new things. Make the other fellow prove his proposition first.

Why be in such a hurry? It usually takes twenty years to whip even a good idea into workable shape; as for example, graphic algebra and trigonometry functions for high school students. The individual's life is short, and naturally he is impatient for results. Also his mistakes as an individual are of little importance to the community. But the life of an institution is long. A delay of two, three, four years means nothing, but a mistake usually is very costly to many people for many years.

Another by-product of this alliance between mothers' club, weak superintendent, and pseudo-psychologist is that the schools are being made the depository for many responsibilities that properly belong somewhere else. Maybe the schools are destined to assume the role of "my brother's keeper" and that a geometry teacher will have to qualify also as a visiting nurse. But I think not. I like to think, as President Lewis implies, that the business of a high school is to train and exercise the mind; that the business of a geometry teacher is to train and exercise the logical faculty and cultivate the habit of work, and I believe his lasting influence is created right in the classsroom.

A visiting teacher from France stressed the point that over there the schools (the Lycees) assume no responsibility—not even that the student will pass. That is up to the student and his parents. The students have no social life; they work. A British professor speaking about our universities said: "We think you dry nurse your students too much." About one thousand students were turned back from one university last year because they could not make the grade. In many of their homes uncomplimentary remarks are being made about the local high school. This is going on all over the land. Sooner or later we will be told to mind our own business and teach school. We are committed to the policy (unique in the history of the world) of free tuition to every one through the university. The burden is heavy and will be heavier. I think we will modify this policy, at least in practice. In fact, our universities are doing so now by setting up standards beyond the reach of many. The high schools will be compelled to fall in line. If a sixteen years old boy is not in school, society expects him to work eight hours per day, six days each week, and fifty weeks a year. Those who get any where usually do more than that. But if he isn't willing to do this much, society gives him little consideration. If he doesn't observe the hours and do reasonably good work, he gets fired, and neither his parents nor his friends think of berating anyone except possibly the boy. But if the same boy or girl is in school with free board, free clothes, free teaching, and elaborate equipment for both work and play furnished by the community, different standards seem to prevail. We don't dare ask him to do eight hours a day only five days a week for even thirty-eight weeks a year. If anyone suggests dropping him or otherwise penalizing him for lack of application or lack of fitness for the job, a loud wail goes up about the defeated boy. I do not believe this double standard can endure.

Instead of dealing honestly with the student and his parents, as any private employer would do, many schools are condoning and camouflaging ignorance and unfitness by such devices as overgrading, diluting the requirements, segregation, etc. They even neglect to define the word *failure,* permitting the notion to become current that failure to pass an examination is necessarily a tragedy, when such is not the truth at all. A boy has

not failed when he stubs his toe and falls down. He has failed when he refuses to get up and do something about it. If he refuses to do this with sympathetic teachers and parents standing by, then it is an open question whether there is much in him that is worth saving. Many a so-called failure really is a very stimulating educational experience, an important step in the salvation of a soul. Sometimes I entertain the notion that the really unfortunate student is the one who never fails to pass.

My teacher of geometry gave me one hundred per cent for the year. Naturally I was sent on to college to become a mathematician. After it was too late (at least, I thought it was), it became clear that I was a very ordinary student of mathematics, not qualified at all to specialize in it. The teacher who thus misled me and my parents was both a Normal and a University graduate, and since then has been the president of two colleges, the second one being one of the choice places of the land. He could make a wonderful speech, but he left a trail of dissatisfied students and co-workers behind him. He simply could not be brutally frank, or careful with details, or deliver as much as he promised.

I tell this story because I believe a great many students have had, and are having a somewhat similar experience. Our school receives a large number of students from other secondary schools —students who for the most part expect to go on to college. Last year we had about forty in third-year algebra who had their first-year algebra in some other school. About eighty-five per cent of them failed to pass our third-year course in their first attempt. This is typical of what happens year after year. Of course we don't consider our standards unduly high. Not all of our graduates make one hundred per cent in the College Board examinations. Each year about fifteen enter the University of Illinois. We find that their freshman averages run from one to two per cent higher than their four-year high school averages. Very seldom is there a spread of as much as five per cent for any one student. This, we think, is about as it should be if the student is to feel satisfied with his high school preparation. We find that a student who comes to us with a *C* rating may be anything from C to—0. I think a great many students are being permitted to believe they know how to work and that they have

a passable grasp of mathematics, when the fact is they never have worked and they "don't know nothin'." In general their reactions to our treatment are not unkind. They are glad to know the truth before it is too late. Sometimes a student's attitude verges on the heroic. For example, a girl entered this year after three years' exposure to a course in general mathematics. She asked to be put back into first-year algebra. Her reason was: "I want to know something about it." There is more tragedy as a result of over-grading than at the other end of the line, though it is the so-called failure or defeated student who has been receiving most of the sympathy.

Under a different administration my brother failed to pass his second-year Latin and mathematics. Father, being wise in his day, had a friendly session with the boy and it developed that he wanted to be a farmer. Within a month he was a farm helper and has been a farmer ever since, contented, influential in his township, and financially prosperous. Sometimes I wonder what would have been his fate had he been compelled by father, or persuaded by some of our modern devices, to linger two or three years longer in high school. Since that episode most of the sob stories about defeated and discouraged students have left me cold. Again I suggest that a student has not failed when he falls down. He has failed when he refuses to get up and do something about it. When a teacher tells me that ninety-five per cent of his geometry students are passing, I don't know whether to laugh or weep. Something tells me he is kidding himself, and his students. He is trying to side-step one of Nature's laws and a few years hence he will have some disillusioned and uncomplimentary critics.

The next important movement in primary and secondary education will be towards higher standards of achievement, with more stress on the work habit and more general agreement that "to him that hath shall be given." We shall quit stepping our geometry down to the ever receding level of second-year intelligence and ask our students to step up to the level of second-year geometry. The universities are busy clearing out the dead stuff so that the live stuff may have a better chance, although I suspect they still harbor many, many students who have very little business there. The best private preparatory schools al-

ready are well cleaned up. The public schools will do likewise; not immediately perhaps, for they are very busy just now juggling I. Q.'s, but bye and bye when they can get around to it. Even the parent-teachers associations will discover that the game of being my brother's keeper, like all other games provided by our Creator for the amusement and edification of His children, must be played with due regard to certain divine rules, one of which reads: "To him that hath shall be given."

Also there is another little rule in this game that amateur uplifters overlook; in fact, it is the very first rule of the game. To be your brother's keeper it is necessary for the other fellow to be your *brother* and not a third cousin or a total stranger. Sometimes it is difficult to distinguish, and if your guess is wrong trouble is sure to follow.

Some good women sent this one to Queen Marie: "We are loth to believe you do not understand the pernicious effects of smoking." No wonder the good Queen went home early. The uproar over the Volstead Act indicates that someone has mistaken an utter stranger for his brother. A manufacturer admitted to me, with a sob in his voice, that many of his employees did not seem to want to be saved. I tried a little segregation experiment with some third year algebra students. Within a month two mothers informed me that I had labeled their boys dumb, therefore they were proceeding to be dumb. In August our principal received a short list of names from the eighth grade authorities with this memorandum. "These students should not attempt a regular high school course. Their parents have been advised with, etc." So a special course was fixed up for them. Three did not report in September, and the parents of three others were on hand to protest against the special course. They wanted their children to take a regular course like other folks. A citizen pleaded for more extensive vocational courses in the high school. When asked if *his* boys would elect them, he replied in substance: "Oh no; but there must be many others who would." Some women asked for more athletic equipment without first making inquiry as to the School Board's plans or the state of the exchequer. This failure to distinguish between a brother and a third cousin caused the irritation in the toothbrush episode. It is the little joker in a popular pastime that

eventually will help to kill the fad and compel the schools to mind their own business.

The average American, be it said to his everlasting credit, is a sentimental fool. Such phrases as "the submerged tenth," "the underprivileged boy," or "the defeated student" make a mighty tug at his heart strings. Even our business men talk about *service* as they separate us from our cash. And they half believe it and derive much pleasure from this thought. Most of us long to be a big brother to some one. We frequently make a mess of it because we are young and new at the game. We don't know the rules and we lack technique. It was only yesterday we moved in from the farm where social problems were simple, a home was a home, and bountiful Nature covered up such mistakes as we made. We have developed no great skill in social diagnosis; in other words, we have not acquired the scientific attitude of mind, even though a varied assortment of pseudo-scientists, amateur statisticians, and one-armed surveyors would have us think otherwise.

Conservative teachers of geometry always have had to contend with fads in their specialty. I wonder if any of you remember the fad for vest pocket geometries? The little text by Wells, say,—very thin paper, limp leather cover, packed with corollaries so-called—about as big as a minute,—looked very much like a prayer book. And that is what the irreverent boys called it. A few years ago, two fads sprung up and struggled for supremacy. One had for its central idea the revamping of the text. In some cases the postulate of parallels was rewritten and followed, of course, by a very radical change in the character and the sequence of many proofs, together with considerable dilution of the whole contents. In others a great mass of experimental geometry was introduced and followed by considerable work in symmetry.

About the same time the fad for corollated mathematics was born. Watched over by very competent and very tireless nurses, it gave promise of developing into a big boy, when some one hit it with a brick. I don't know who that person was, I wish I did, for certainly he did a skillful job. He simply called our attention to the essential difference between the chemical union of things that have a real chemical affinity for each other and the

mere dumping of unlike things into the same barrel. Geometry teachers everywhere saw at once that much we consider worth while in geometry has no chemical affinity for anything else in the high school course.

The situation during those years was complicated further by the birth of the theory that there is no transfer of discipline. This caused many schools to drop geometry from the list of required subjects and others to dilute the course.

These fads have subsided. The opinion now seems quite unanimous that the values we hope to extract from geometry are best realized by preserving its identity and presenting it in something akin to the traditional order. Of course irreparable damage has been done. In many schools a generation will have passed before geometry is reinstated in its proper place of importance. It will be many years before a corps of teachers can be developed who have been trained to teach geometry. The training schools have neglected that phase of their work. I doubt if half the geometry teachers in the state would know a nine-point circle if they saw one. I know of only one textbook worthy of the name that has been written expressly for teachers of geometry in the recent past. This great body of poorly trained teachers, not being sure of their ground, are easily influenced by every half-baked theory, every ill-considered bit of propaganda that comes to their attention.

It was my privilege to work for six years with a really great teacher of geometry and Latin, Principal Chester T. Lane, Ft. Wayne, Indiana. At Purdue University they used to say they could detect Mr. Lane's students in a class of one hundred by the way they talked mathematics. He taught about thirty-five years in Ft. Wayne and died there. During the last years of his life he received many letters of appreciation from successful men. They all said: "You taught me how to work, how to read the printed page, and the joy of achievement."

Students seldom came to his classes unprepared. He commanded the respect and admiration always accorded the man who really knows his business, but he did not depend upon gumshoe methods to get results. He could make a lazy boy shrink to the dimensions of the proverbial knot hole with his withering

sarcasm. And he did not hesitate to keep them after school until the street lights were burning.

Once I had a student, a girl from a prominent family who always was discouraged with her geometry. I did the usual thing; I tried to bolster up her courage. Finally she asked Mr. Lane if she could drop geometry. Without hesitating, he replied, "I think you might as well; you are not doing much with it."

Which, of course, was not what she was looking for. She decided to think it over.

That peculiar ability to uncover a student's trouble with one remark or question is the unfailing mark of a teacher. Mr. Lane had it to the nth degree. He was fond of telling this old one. Miss Jones was finishing her first year of teaching after graduating from Normal School. The school directors were making their annual visit. Miss Jones asked Johnny:

"How much is 2 x 3?"

Johnny sat like a knot on a log.

"But surely, Johnny, you know how much 2 x 3 is?"

"Sure I do, but you haven't developed me yet."

He remarked frequently that teachers usually underestimate rather than overestimate the ability of a student. The teaching problem is to get them to work. He believed that his job was to teach them to work and to read the printed page before they were twenty years old. After that they would take care of themselves. And he believed that old-fashioned geometry and Latin were first rate mediums for that purpose.

He was the first teacher in Indiana to use the so-called "suggestion method" of attack. He pronounced it the only worthwhile forward step in twenty-five years. He reviewed a manuscript in which the "plan of attack" was the big idea. He was strong for that feature because it had been the focal point of his own teaching for years. That was almost before Mr. Hart was born.

Mr. Lane's chronic complaint was that authors and speakers were forever submitting their stuff to the public before they had been all around, over, under, and through their subject. He

would remark that the Christ himself with the tremendous head-start of divine birthright spent thirty years in obscurity preparing for an eighteen-months' work.

I thought of that remark many times the past summer as I reviewed a couple of new geometries and read the current geometry fiction in preparation for this talk. A college president admits with a smile that he no longer believes various things that he published in a textbook ten years before. The author of a geometry tells me his publishers did not allow him time to proof-read the last quarter of his book. Another thinks that the altitudes of an obtuse triangle meet at a point! Most of them don't know the definition of a converse theorem—at least they follow their so-called definition with a theorem that swears at it. They offer *st.* as an abbreviation for *straight*. Preceded by the digit *1*, it becomes *first* to many students. They say a quantity may be substituted for its equal in any mathematical expression, so the student substitutes *AB* for its equal *CD* in the mathematical expression "*CD* is parallel to *XY*." A teacher of teachers had not observed that a fractional exponent is an operation symbol and not a number symbol; therefore it is asking a good deal to *assume* that one can do to it the various things one does to a fraction. There are two mistakes in the spelling of my name in the program of this meeting. These are little things but they represent the difference between a scholar and something else. Everyone is in a hurry to get his stuff on the market in time for the fall trade, and the accepted way to get your copy proof-read is to let the ultimate consumer do it. Last spring four of us made a critical review of a new text and we found more than 150 things to criticize. Some of them, of course, represented differences of opinion between the author and us and should not be counted, but most of them were glaring errors in technique or type setting. Even our most sacred number π had been mistreated. The bookman said by way of excuse: "Most teachers are not so critical as you people." I wonder why not? We are supposed to be teachers of mathematics. Compare some of these new texts with the Saturday Evening Post and blush with shame. I suggest that we forget the new stuff for five years and devote ourselves to putting the old stuff in shape to teach.

Many of the articles in the mathematical magazines are more harmful than helpful to the young teacher. For example, one writer offers ten reasons why students fail to get geometry and neglects to mention the two important reasons: namely, that the student is not old enough or he does not work hard enough. My little girl cannot do geometry even if she does have an I. Q. of 150 (if you believe in I. Q.'s) for the same reason that that I cannot lift one thousand pounds. Of course we could fix up some foolish little course in paper cutting and folding and call it geometry, but that would not make it so.

When we came to the first test for congruence I spent ten minutes by way of preparation; then I said, "Be able to reproduce this page just as the author has it. If you can not do it at class time to-morrow, you will have to do it after school." Now that assignment is not beyond the ability of any real second year student. Some of the letter-perfect ones said they did it in fifteen minutes. Twelve had it letter perfect. The other ten came back after school and were told to study until they thought they could do it. Within about fifteen minutes all thought they could write it out; and seven of them did. That gave me a rough outline of my job with that class. Probably twelve students who will take care of themselves; seven more who could if they would; and three who are too young. (Since then I have changed my mind about one student.) The real teaching job is to make the seven loafers work. Time spent on the three immature ones is largely time wasted and savors of cruelty to dumb animals.

Every phrase maker is calling this the age of stadia. I would say stadia, statistics, and surveys—the age of the three S's, following the age of the three R's. One surveyor has listed two or three pages of *objectives* (they love that word) for the poor geometry teachers to shoot at, and modestly admits that the list is far from complete. A waste of time, especially since he omits the most important objective. Let your first objective be to make the loafer work or get off the job. Follow that by the four or five other objectives that must occur to anyone and forget the expert surveyor. Plane geometry does not need an alibi any more than English or civics.

A statistical fiend has made an elaborate study of the relative achievements of boys and girls in geometry in New York and

another writer argues for the segregation of the sexes for the study of mathematics. Now everyone knows that secondary sex characteristics are hopelessly mixed up in most of us. We are intermediate types. Many a boy is far from being all-male, and many a girl has the flat chest, or the narrow hips, or the voice of a male. If this confusion carries over into the mental makeup where do we get with these statistics and this segregation theory! I merely say "if," and suggest that may be several preliminary questions ought to be studied. Possibly our best students *in general* are intermediate types, some boys, some girls, because that type conforms best to the routine of school life. My point to-day is that it isn't safe to believe all you read, even in a mathematical journal. Don't embrace the new theory too hastily or too ardently. The professional investigator has to produce something every once in so often or lose his standing. He probably will not believe half of it himself five years hence. At least sneak around and observe how his own children react it.

Of course, some teachers are so poorly equipped for their work that even the experts can't harm them. I have seen teachers of algebra who could not solve a quadratic equation if the coefficients were literal. A half dozen others have asked me to explain the $2 = 1$ puzzle. One man who was being paid to teach algebra, boasted that he told his students "They could not add cats and dogs." I wonder what became of his cats and dogs in multiplication.

The development of a human being should differ materially from the development of a rose or a race horse. The human should become self-sustaining. In this day and age that means he must learn to work, and there is good evidence that this must happen before he is twenty years old. Don't depend too much on arousing the student's interest. All of us have to do things just because they are a part of the day's work. If a teacher keeps this objective uppermost, he probably will earn his salary and need not worry about the passing fads. We Americans have upset various man-made hypotheses and profited by so doing, but the Sermon on the Mount is something else.

Finally, if you ever feel real proud of the results you are getting, I know a fine little cure. Just ask your students to define a *plane*. Nothing fancy. It will be sufficient if they can

say, "A plane is a flat surface." Even the few who can say that much will harbor the delusion that the plane surface is the only type of surface there is. If you are a real glutton for punishment you might ask them what a *degree* would look like if they met one on the street.

A kind visitor said to me, "Up our way we usually serve a meal in courses. First a bit of something to whet their appetite for the feast. Then we carve and serve the main dishes on beautiful china and silver. At the end we recommend a nap or maybe a stroll around the block to ward off indigestion. But your students seem to have developed an appetite by chopping wood, and they serve themselves, for the most part, in the good old country fashion. After the meal they stroll back to the wood pile."

Of course my students don't do any such thing, but his idea was so exactly in line with my thought to-day that I am passing it on to you.

MATHEMATICS AND SCIENCE

An Association for Mutual Benefit

By HARVEY A. NEVILLE
Department of Chemistry, University of Illinois

There is a story, probably apocryphal, illustrating the celebrated loquacity of President Coolidge. He is said to have attended church one Sunday morning unaccompanied by Mrs. Coolidge and to have been catechized by her on his return as follows:

"Did you enjoy the sermon?"

"Yes."

"What was the minister's subject?"

"Sin."

"Well, what did he say about it?"

"He was against it."

When a chemist attempts to talk about mathematics you may say that he is as much out of his proper field as the preacher is when he discusses sin. It may be that both lack a practical knowledge of the subjects, in that they both maintain a strictly amateur standing.

I was assured by the program committee that the secondary school teachers of mathematics are anxious—or at least willing—to co-operate with the college teachers in the sciences in improving the preparation of the freshmen who come to us. I was told that the mathematics teachers, realizing the importance of mathematics as a tool in science and practical life, desired to know what mathematical ideas and principles we consider most essential, what deficiencies we find in the preparation of students in these matters, and what suggestions we could make for the improvement of the situation.

The average man may be dimly conscious of the appeal of higher abstract mathematics to some minds. He may realize that in it they find a beauty akin to the beauty which most minds seek in art and music. But, I believe, to most people mathematics is only of practical interest, a means towards certain ends, a tool for the purpose at hand. In this sense it is the controlling factor in álmost every undertaking of a practical or

scientific nature. Mathematical reasoning is the only true conservative influence in the world today. It keeps us from too radical ideas in all fields of thought. Bolshevism and perpetual motion and sometimes matrimony—all alike succumb to mathematical calculation. It does not follow, however, that professional mathematicians are always sober, conservative people.

As I understand it, the stated aims in the teaching of mathematics are *practical, disciplinary,* and *cultural* in nature. I believe we agree that these are not mutually exclusive or separate aims but are rather conditions to be fulfilled simultaneously. It would seem possible to teach mathematics as a purely practical subject and at the same time demand neatness, clearness, exactness, and other good mental habits. Furthermore, I believe that the cultural aim will be more nearly realized by an intense emphasis on the concrete nature of the subject. You will agree that there is more of cultural value in matters which we can comprehend and apply than is indicated by a parrot-like ability to converse superficially concerning that which we know not. In other words, let the student learn first the practical use of mathematics and all those other values will be added unto him.

If then we admit that the practical aim of mathematics is of prime importance should we not determine what principles or ideas of mathematics are to be of most use to the student both in the pursuit of his education and in practical life? Should we not ask the man in the street what mathematical knowledge he has found useful and even indispensable and then agree to emphasize the training of our present students in this particular knowledge?

Fortunately the actual necessities are few. I can briefly but truthfully indicate the mathematical need of the freshman in elementary science by stating that if he is able to solve arithmetical problems involving *proportion* and *percentage* his equipment will prove sufficient. I believe that this statement will also apply directly to the mathematical need in everyday life. After all, science is merely a method of procedure and all people are scientific to a greater or less degree—though many would indignantly deny this. Someone has suggested that a scientist is a person who has his typewriter equipped with a ? on *both* shifts and a period on only *one*.

I do not know whether any of the freshmen we have in chemistry can solve simultaneous equations or prove the identity of two geometrical figures, nor do I know when if ever they will have an opportunity to use this knowledge if they happen to possess it; I do know that an alarmingly great number of them are unable to analyze simple problems in arithmetic involving proportion or percentage, as I intend to show you in a moment. How important is this? It is so important that a student is unable to make a passing grade in the chemistry course until he acquires this ability, and I believe the same will be true in physics. On the other hand, I believe that every instructor in freshman chemistry in the university will agree that if the student comes to us with the ability to reason arithmetically we can readily teach him chemistry.

Because we realize this, we have found it expedient to devote the first quiz hour in the beginning courses to instruction in arithmetic and at that time give a qualifying test consisting of a few simple problems. From the student's ability in arithmetic we can predict his performance in chemistry and can give particular attention at once to those students who show a handicap or poor preparation in mathematics.

Is it not true that ratio and proportion are taught in arithmetic, in algebra, and in geometry? Yet many students who have "had" all three have no ability to use the principle of proportion and do not even recognize it on sight. The student admits that $3/4$ is a fraction but has never realized that it is a ratio. He admits the truth of the equation $3/4 = 6/8$, but has not thought of it as a proportion. He cannot tell us that per cent means ratio to 100 and that hence percentage problems are really problems in proportion in which one ratio of the proportion involves the number 100.

I shall not bore you with statistics but I wish to show you the type of problems which freshmen can't solve and to indicate the grades made on these problems. You will find it difficult to believe that college students are so deficient in this respect, but the test papers are on file in the chemistry building. Here is a typical set of test questions:

Aptitude Test

1. 16 is what per cent of 25?
2. Solve for X: $\dfrac{16.4}{4.8} = \dfrac{5.5}{X}$
3. If a copper ore contains only three per cent copper, how many tons of the ore must be worked in order to obtain a ton of pure copper?
4. If Champaign is 120 miles from Chicago, what is the distance in kilometers? (A kilometer = 2/3 mile.)
5. If 27 pints of grape juice cost $5.40, what is the price of grape juice per gallon?

From 20 to 30 minutes was allowed for this test. A few other problems which have been used are:

1. If 400 gallons of water flow through a pipe in 9 hours, how much water flows through in 4 days?
2. There are in a room 30 women, 18 men, 15 children. What per cent of the group are men?
3. In a certain chemistry class 68% passed; if 48 students failed, what was the total number of students in the class?

Please note that this test was given after about one-half hour had been spent in recalling the principles of proportion and in reviewing problems of the same type. The students were not taken unawares and had sufficient time to orient themselves to the subject. We usually start with the geometrical representation of proportion to illustrate the true significance of the principle. The average of a section of 22 students was 55%. Some sections were better and some were worse so this may be taken as representative of the 700 students. Many were given a grade of 0 on their results. Considering the problem on the distance in kilometers from Champaign to Chicago:

Result:	180	200	160	153	80
Obtained by:	12	2	1	1	6

The errors were not mechanical but in reasoning.

But these students are here in the university, many on their own resources and all managing their own financial affairs which must present many problems of more difficulty than those I have just read. What is the difference? I believe that in their own affairs they see the meaning of the problem and are able to think it through logically. They have not been trained to do this in regard to a problem in a book or on the blackboard. The ability must be there. I do not believe that students who

lack the reasoning power requisite to these problems could ever reach college. May it not be due to emphasis in their training on the mechanical processes involved in problems rather than upon the meaning, the analysis of the problem. And perhaps also there has been insufficient practice in the use of the reasoning faculty and hence a lack of its development. I believe this is particularly likely to be true in the teaching of algebra, where, as I remember it, emphasis seemed to be placed upon skill in the manipulation of letters and exponents with no thought as to their possible meaning or application. As a matter of fact, some of our students try to balance chemical equations as if the chemical symbols were algebraic!

If logarithms are taught or even mentioned, the mechanical application of logarithms in the slide rule should be presented. Rather, I should say, enough of logarithms should be taught to introduce and make clear the principle of the slide rule, and students should be given practice in its use. This labor saving instrument is not yet sufficiently appreciated, though its use in business, (banking, life insurance, etc.) is becoming as general as in scientific work.

It seems appropriate to say a word here for the metric system of measurement. This is a matter supposedly taught in most schools, yet students come up without a clear conception as to whether a meter more nearly approximates a mile, a yard, or an inch. I have supported metrics for educational reasons principally because that time now spent in memorizing the complicated relations of units in the English system can be utilized to teach students to *use* measurement in practical problems. The relations in the metric system also furnish practice in decimals, a subject in which students are often deficient in spite of our decimal monetary system. The official adoption of metric units in all countries except the British Empire and the United States, and wide usage in these two countries for scientific and other special purposes make a working knowledge of the metric system a practical necessity in modern life as well as a cultural advantage.

May I be permitted to say that elementary mathematics may be considered essentially an experimental science—the science of measurement? Yet I am impressed by the fact that the method

of procedure in geometry, for example, is exactly the opposite of that used in chemistry. In the latter we perform an experiment, make observations, draw conclusions, and, if possible, derive a generality or law from our data. In geometry, however, the first thing that greets the eye is the "theorem" or "proposition" which is already the general statement or law which you hope to discover by a circuitous route arriving where you started. The subject is thus taught in reverse order of the way in which its truths must have been discovered. Modern science, on the other hand, owes its enormous progress in recent years to the application of the inductive method and is, or should be, taught in this logical manner.

Might it not be possible, and perhaps desirable to some extent, to convert geometry into an experimental science? How many circles can be drawn through three points not in a straight line? Can the student not discover the answer for himself? Would it not be a more stimulating exercise than to memorize the proof that "all circles which are drawn through the three points coincide and are hence the same circle?" Such proof, to my mind, is mere dogmatism, a remnant of medieval philosophy, and subversive to originality. The fact itself, not the proof, is important, and is worth more to the student who discovers it for himself. Suppose a physical scientist should write a *laboratory manual of geometry*—imagine such a book and you have my idea of a valuable contribution to the science of education.

Do not understand that I am opposed in general to the teaching of abstruse geometry, quadratic equations, the binomial theorem and other matters dear to the heart of the mathematics teacher. Not so! But I am opposed to attempting to teach these before the minimum *practical* essentials of mathematics have been surely and forever grounded in the minds of the students. I repeat, it is difficult to understand just what value for culture or for training may be derived by a student from an advanced course in mathematics if he does not understand simple fundamentals. For this reason there should be a distinction in the material presented in any course, at least in the mind of the teacher, between that which is to be of later use—that is, essential in a practical sense—and that which is of value only for general culture or an effect in training. The material of the

first type should be emphasized, applied and, if necessary, repeated *ad infinitum*. This would probably mean a segregation of the quick minds from the slow. Students who do not show a mastery of arithmetic should not be allowed to proceed to more advanced courses. It is becoming more and more recognized that the student should be in competition with individuals of his own degree of ability. This is for the sake of both types of students. One of the most outstanding faults of our American type of education is that we do not do justice to our best students. In a mixed group the best students find the work too easy, the pace too slow, the teacher has no time to interest them, they lack the stimulus of sharp competition and often drift into mediocrity. The plan of segregation is being tried partly or completely in many places and reports as to its success are now available.

The deficiencies I have been talking about began in many cases, as you well know, before the students reached high school. There is not sufficient opportunity in high school or in college to correct all of these. That is why so many college graduates are practically illiterate. Educating the youth is like building a chimney—it must be from the ground up, not from the top down. It should be broader at the base than at the top. If there are any trimmings, they should be at the top and added only after the useful features of the chimney are complete. I shall refrain from remarking whether or not the chimney should smoke when completed.

INTEREST OF PUPILS IN HIGH SCHOOL MATHEMATICS AND FACTORS IN SECURING IT

By ALFRED DAVIS
St. Louis, Mo.

A few years ago attention was attracted to the high percentage of failures among pupils taking high school mathematics. Sometimes as many as 50% or even more would fail in a single class. A little consideration would have convinced the teachers that such a situation must soon attract unfavorable criticism, and that this might be expected from those who were not most favorably disposed towards the subject. At a time when every subject was to be tried and judged, not according to its past achievements, nor according to its future possibilities, but according to present status alone, someone was certain to take a one-eyed view of high school mathematics and condemn it as an unsuitable subject to be required of all high school pupils.

One of the hasty charges made was that mathematics was a bore to high school pupils,—that they disliked the subject, and that the natural development of pupils' powers was hampered by attempting to drive, or drag them through the dull drudgery of the subject. Teachers who had been engaged in the class room doubted the truth of the charge; and so carried out several investigations to determine its truth or falsity. Mr. Raleigh Schorling, then of the University High School, Chicago, investigated three high schools in and about Chicago, to determine whether pupils found much enjoyment, a little enjoyment, or no enjoyment from the study of mathematics. Representing the schools as 1, 2, 3, the results were:

	Much enjoyment	A little enjoyment	No enjoyment
1	50%	41%	9%
2	52%	40%	8%
3	46%	43%	11%

In each of these schools one or two years of mathematics was required of all pupils. Out of a total of 2,018 pupils 88% said they found some enjoyment in mathematics, 49% said they found

much enjoyment, and only 12% said they found no enjoyment. Any subject ranking higher in the preference of the pupils was elective.

The writer secured answers to the following questions from 3,105 pupils from four of the high schools for white pupils in the City of St. Louis, Mo.:

Age--------- Grade--------- Boy or Girl (underscore).

1—Do you enjoy the study of mathematics?

(underscore) Arithmetic,—Much—A little—None.
Algebra,— Much—A little—None.
Geometry,— Much—A little—None.

2—Why do you study mathematics?

3—What advantages do you expect from its study?

4—Of all your high school studies, which do you enjoy the most?

5—Which of your various studies do you value most? Why?

It is not my intention to take you through the details of that report at this time, since a summary of it may be found in The Report of the National Committee on Mathematical Requirements p. 533; suffice it to say that the results are somewhat more favorable for mathematics than the Chicago results, as the following table shows:

The measure of enjoyment realized in their mathematical studies by the students of five St. Louis high schools.

	Number of Pupils		Boys			Girls			Total		
SUBJECTS	Boys	Girls	Much	A Little	None	Much	A Little	None	Much	A Little	None
Arithmetic ---	1564	785	65.7	30.0	4.2	61.3	30.7	8.0	64.2	30.2	5.6
Algebra -----	1785	915	59.5	33.2	7.3	67.1	26.1	6.8	62.1	30.7	7.1
Geometry ----	1106	479	57.6	34.6	7.8	47.6	40.7	11.8	54.5	36.5	9.0

Mr. Walter F. Downey of the English High School, Boston, investigated the mathematical interests of 6,978 pupils of 15 high schools, representing 8 states, scattered over the country. In answer to the question, "Do you like mathematics?", 84% answered "yes," 14% answered "no," and 2% answered "partially." In one of these schools mathematics was required of all pupils for four years, in others it was required for less time, or in some courses of study and not in others. Mr. Downey found no marked differences on the basis of sex. He concluded that

mathematics ranks well on the basis of preference; it is liked by more than four-fifths of the pupils who study it; pupils like mathematics on the basis of general interest, usefulness, and mental training, while they dislike it (when this is the case) because it is difficult. This report may be studied further in the Report of the National Committee.

It appears that the reaction of high school pupils towards mathematics remains much the same regardless of whether the subject is required or not. This is probably due in some measure to the variety of factors that lead them to elect the subject. The writer believes the attitude of the student body is much more important to-day than when mathematics was required of all pupils. With this idea in mind he conducted an investigation with the 135 members of his classes in May, 1926. The results are very interesting and we give them for your information and with the hope of giving inspiration and encouragement to teachers of mathematics. No attempt was made to prepare the pupils for the questions, except that they were informed that the answers would have no effect on the grades for the term. The classes with their numbers were:

Class	Number
Second half year of algebra	28
Third half year of algebra	31
Second half year of plane geometry	31
Solid geometry	28
Trigonometry	17

Why did you elect high school mathematics? (Some gave several reasons). Answers:

Parents or others advised it	19,	or 14%	of pupils
For college entrance requirement	42,	" 31%	" "
Preparation for vocations, and use in other subjects	74,	" 55%	" "
Culture (for clear, accurate and rapid thinking, to complete education, etc.)	53,	" 36%	" "
Like the subject, interested in it, etc.	38,	" 28%	" "
Poor in arithmetic, and so needed it	3,	" 2%	" "
As part of course (two courses require it)	17,	" 13%	" "
Advised not to take it to avoid failure	1,	" 1%	" "

Several questions were asked to determine what the pupil thinks he is getting, or expects to get, from the study of mathematics. Answers:

Cultural advantages	About 75% of the pupils
Vocational needs, or college entrance	About 25% of the pupils

Some gave both as the expected results.

Do you think you have gained from its study anything not derived from other studies? What?

About 95% answered "yes" to the first of these questions. To the second they gave, "ability to think for myself," "logical thinking," "accuracy," "know when I am right," "ability to attack and solve problems," "necessary to study science," etc.

What pupils, if any, would you advise to take mathematics in high school? Why? Answers:

All pupils	61%
Those who need it in vocations and for college	18%
Those who like it or are interested in it	18%
Boys only	3%

The answers to these queries are inspiring to any teacher of mathematics. Since they tell some of the things I would wish to say, I shall let each class speak for itself. The answers are representative and some of them were repeated many times, in thought, if not in words.

Let us hear first from the class in second term algebra. The boys will respond first:

"If one did not take mathematics in high school, he might not be able to fill the position he desires. He might be cheated by others and not know it."

"I expect mathematics to help me solve problems in business. If my associates in talking happen to use algebraic terms, I will understand what they are talking about."

"I have found that in solving problems, one must have a clear brain and be able to concentrate. While a person could not do his mathematics after a hard day's work, he could more easily do his English, Science, etc."

"I elected mathematics,

1. Because of its wonderful history. Men, brilliant men, took it as their vocation. Surely there must be something in a subject that men, noted for their wisdom, have tried to perfect since earliest times.

2. Because of its value as a problem settler. Men as problem settlers make history.

3. To develop my mind. To make me think of the 'what's' and 'why's' of things."

"I will have more tools to work with, and more to think about."

"I have learned that mathematics is useful, and not taught for the sake of teaching."

The girls will respond next:

"I expect to become accurate from the study of mathematics. I can see already a change in my thinking. It also enables me to think faster and more clearly on other subjects."

"A person might have exceptional ability in mathematics, and if he did not take the subject in high school, he might miss his proper life work."

"The poor students should take it to improve their minds."

"A person not taking mathematics in high school would limit himself in the vocations from which he could choose."

"One of the chief fundamentals of an education is mathematics."

"I have learned to think things out from the beginning. In other subjects the facts are given and you learn them, but in mathematics you must use your brain and think them out. In mathematics you must be accurate, while in other subjects you need not be so accurate."

The boys in plane geometry respond,

"I would advise all those who do not express themselves clearly in other subjects to take mathematics, for it teaches you to think clearly, and through that to speak clearly, more rapidly than any other subject."

"A person who has not taken high school mathematics would be apt to accept as facts what was told him without thinking for himself."

"I have learned to think before making a decision, and then to stick to it if I am convinced that I am right."

"Before advising anyone to take mathematics I would ask him if he liked the subject. If he did not I would ask him if he thought that he would determine to like it, and would work to bring about that end. A person must like a subject to get the most out of it."

"The problems or the theorems are not the most important in mathematics, but the effect of the thinking is what should last."

"When I debate, which is quite often, I have learned from mathematics not to make assertions that I cannot back up. I have also learned to arrange my arguments logically so that they hang together, and one seems to grow out of the other. I would especially advise the active members of the debating society to take mathematics for they would gain by it."

"I would advise everyone to take mathematics in high school, but particularly those who hesitate in voicing their opinions, and are not sure of themselves. I have gained more confidence in myself."

The girls respond:

"I do not think one's education is complete without mathematics. Without it one would not have the satisfaction of studying a subject whose principles never change.

"In mathematics you can never dodge the real issue. You must have words at your command to express exactly your meaning. You can't 'play around' saying one thing and meaning another."

"Pupils do not get as good a chance in any other field to stand up firmly behind a proposition and know they are absolutely right."

"One would never regret that he had not taken mathematics for he would not know the advantage to be gained through it."

"I would never advise a pupil to study mathematics who says before he starts, 'I do not like it and I know I will never pass,' for he will never get it until he gets those thoughts out of his mind."

"Anything learned from mathematics is always true, whereas in history and other subjects, statements are changed and varied a great deal."

"I have gained the knack of thinking more clearly and of expressing what I have in mind, in my own words without getting muddled."

The boys in solid geometry respond:

"Without mathematics one will not understand what other people about him understand. He is likely to look upon out-

standing public questions and other questions in a narrow bigotted manner and will not understand his neighbor's more liberal point of view."

"I think that mathematics, especially geometry, has forced me to become more accurate and practical, not only in mathematics, but also in other studies. It has taught me to examine things carefully from all angles; and to study diligently those things which I do not understand, but which can be mastered by study."

"Mathematics has taught me to be exact, to think out difficulties and to think more about my surroundings."

"If one did not take mathematics in high school I think he would not be popular in social or business circles, because he would have no knowledge of many items of discussion."

"In physics a special class was made up of those not sufficiently prepared in mathematics, otherwise they could not have carried the work."

"I think mathematics has helped me to use correct English, for if one puts a wrong word into a statement or proof, it spoils the whole thing."

The girls respond:

"Mathematics has been of great value to me because it has taught me to try to think clearly and concisely having a reason for all I do."

"I elected mathematics because my brothers and sisters advised me to do so. After one year I continued on my own initiative."

"I have learned to think things out. Not to jump at conclusions, and not to judge by appearances only. Mathematics has enabled me to think more quickly and clearly. It has given me a broader view of many things and enabled me to talk more intelligently."

"If I were thinking of employing a high school graduate, I would not consider him if he had not taken high school mathematics, because mathematics would make him brighter and quicker to act and to understand."

"I would advise everyone to take mathematics. Even if they failed they could not have been in a mathematics class one or more terms without getting some benefit and understanding a few of its principles."

The boys of the third term algebra respond:

"Mathematics has taught me to beware of saying things I cannot prove. It has taught me how to go about proving a statement and what a proof is. I would advise students who like to be sure of what they are saying to study mathematics."

Values of mathematics, to me:

1. Necessary to a well rounded education.
2. It forced me to be accurate.
3. It taught me to systematize my thought and actions.
4. It taught me to reason on a sound basis.
5. It gave some insight into the boundlessness of certain things.
6. It showed me that mathematics is related to almost everything.

"Mathematics requires the best of English. It has an interesting history and it is a language in itself."

"I had been told at grammar school to beware of mathematics in high school because I would never pass."

"I can honestly say it is more important and more valuable than any other subject I have taken or could have taken."

"I don't believe a student deserves a diploma from high school if he has not taken any mathematics."—President of the Senior Class.

"My older brothers and sisters had all taken mathematics and found it of value, so they told me to take it."

"If one did not take mathematics in high school he would miss one of the best things offered."

"If one did not take mathematics he would be ignorant of the far-reaching influence of the subject, and he would be helpless before problems that often confront us in life."

"We learn to prove some of the facts that are merely taken for granted by uneducated people."

"One who has not studied mathematics is likely to be seriously handicapped if he seeks a position of any particular prominence."

"I have gained the idea of working at a problem until I get it."

"I think the one not taking high school mathematics would be at the mercy of those who wish to take advantage of the ignorance of others."

"I thought that all people should know some mathematics besides arithmetic. I do not believe arithmetic is sufficient for

anyone who has ambition to get on in the world. I believe mathematics has given me a greater insight into life and its problems. Other subjects are not very definite, whereas mathematics is absolutely exact. You get it or you don't get it. I believe it is part of a man's job to know mathematics."

"I think mathematics has made me more definite and accurate. It has given me a clue to solving problems outside of mathematics, setting down the problem, what I know about it, and what I want to find out."

"The power of reasoning is, I think, gained more from mathematics than from other subjects. In other subjects there is more memorizing."

"I have gained a knowledge of the superiority of mathematics over other subjects, because I have seen that practically all the progress of the world (in science, etc.) is being accomplished by the aid of mathematics."

"I was advised by people out of school to take mathematics. These people realized its importance in commercial occupations."

"The value of mathematics is very great to me. It enables me to reason better, to know how to derive formulas, and to think before I speak. It has taught me to think rapidly, and to speak with straightforwardness and with accuracy."

The girls respond:

"Gains from mathematics,
1. Logic.
2. Broadening the mind.
3. Quicker methods.
4. Makes one equal to the demands of the modern world."

"Mathematics has taught me to say things briefly."

"I had watched other pupils do their work and I became interested."

"If one did not study mathematics in high school he would never know the actual value of mathematics in one's life."

"It gave me an opportunity to know what some of the terms mean that I hear from day to day outside the school room."

The boys of the trigonometry class respond:

"I would advise any sensible student to take mathematics in high school, because:

1. It is good training for the student who likes to work things out for himself and find the how and why. He will not find it very difficult.
2. The person who memorizes, without getting the meaning, and is able to recite perfectly and get high grades, is made to think.
3. Mathematics is used in everyday life as well as in schools and laboratories.
4. Mathematics teaches system and precision."

"Mathematics has enabled me to understand many things I come across in newsapers, magazines and books. It has aided me greatly in science."

"Those who do not intend to work should not enter high school at all, all others should take mathematics."

The girls respond:

"Mathematics was not extremely easy for me, but I felt I had accomplished something when I had worked a considerable time on a problem. I would not consider my course complete without it. It has taught me to think for myself."

"All subjects give you knowledge, but mathematics cannot be learned from a book, it must be thought out."

"I elected high school mathematics because my parents wished it. My mother was a teacher of mathematics and realized its value. However, I planned to take as little as possible, but after taking algebra I decided that a major in mathematics would be much more advantageous and likeable than in history or a language."

"In French, history, etc., one memorizes and absorbs bare dry facts with no real thinking, but mathematics is just the opposite, one memorizes to a degree, but most of the work is thinking out problems for yourself."

"I would advise pupils to take some mathematics in high school. There are two types of pupils, one naturally good in mathematics, and the other just as naturally dull. To the first the subject is a pleasure and a real profit; he learns how to think quickly, how to study, and he gains real knowledge; for the other it awakens his mind, and even if he doesn't shine in the subject, he gains more than he probably realizes."

We pass these quotations without further comment, since they themselves show the student attitude towards mathematics.

Let us now consider briefly some of the factors which may be made to contribute to a satisfactory response from pupils towards mathematics.

High school mathematics is capable of making a universal appeal. It should be the business of the teacher and of the mathematician to see that this is accomplished. The entire community should be made to recognize and feel the importance of the science. Practically all the sciences, as they become more exact, demand its services, for stating in concise terms the results of their investigations, and in showing relationships between theory and observed facts. Without it civilization would revert to barbarism, the wheels of industry and commerce would cease to move, and the universe would shrink from almost infinite proportions to a narrow cell.

It is especially important that girls should recognize the importance of mathematics and elect it as a study. The number of girls in our classes decreases with the more advanced courses. The number would be apparently much smaller were mathematics not required for admission to many of the colleges. Yet women are daily becoming more important in the political and economic life of the nation; they are becoming a prominent factor in industry, in business and in the professions. As the directors of homes they occupy a vital relationship to the education of both the boys and the girls of the next generation. It is especially important that all prospective teachers study the science. Yet since it has been made elective in St. Louis, a considerable number of young women preparing to teach in the city have entered the Harris Teachers College without high school mathematics: in Sept., '21, there were 5; Jan., '22, 12; Sept., '22, 11; and Jan., '23, 10. The records of this institution show that those who had had high school mathematics received the higher grades. In view of the fact that grade teachers may eventually become teachers in the high schools the seriousness of the matter becomes evident.

The pupils in our classes are not properly graded to gain the best results. High schools have been growing rapidly in numbers, and this condition of growth is likely to continue. The result is that the average of mental ability in our classes has been lowered. The high per cent. of failures is largely due to this; for some teachers have tried, but without success, to maintain high standards of scholarship, and pupils have been sacrificed for these standards. On the other hand, some have tried to

meet the situation by eliminating all the really difficult, and, for the better pupils, the most stimulating, parts of the subject matter; other teachers have tried various other devices. As a matter of fact the average class presents an impossible problem. On the one hand we are neglecting the best pupils, and we are inculcating lax mental habits in them; on the other hand we are failing to reach the dull pupil, and we are driving him out of school. As a result we are likely to damage the interests of mathematics in the school and in the community. Probably a remedy may be found in technical and commercial high schools, where the practical, or manual-minded, pupil may work to advantage. This would permit us to maintain the strength of our academic courses.

Another very important factor to be considered in successful teaching is the preparation of the teacher in subject matter. The teacher's mastery of the subject should go far beyond that which he actually teaches. The minimum for the teacher of high school mathematics should include the elementary calculus. I remember, not so very long ago, when the writer said this to the Dean of a School of Education in a great university, the Dean threw up his hands saying, "Why? He is not going to teach the calculus." It is such attitudes on the part of some of our administrators that embarrasses the interests of mathematics. Again, persons are frequently assigned classes in mathematics when they have no desire, and no preparation, for teaching the subject. The subject a teacher attempts to teach should be his specialty and he should love it more than any other in the curriculum.

Not all who master a subject have the ability to teach it. However, most teachers are made, not born. Teaching is a profession and presupposes actual preparation in the art of teaching; not that all methods and proceedures can be "cut and dried" for the prospective teacher, but what he learns serves as a starting point, beyond which he invents and follows his own methods.

Above all, the teacher who is not in love with our youth, who is not thrilled with their eager, earnest expectation and ambition, and who does not take as a sacred trust the responsibility of permanently impressing these young lives, has no place in the

high school. Without these attitudes, without scholarship, and without skill he cannot command respect and attention.

The cry comes from all quarters that pupils do not feel sufficient responsibility with respect to their work. They seem to have heard the philosophy, "I am made to do what I like to do,—what I am interested in." They lose sight of the fact that we may learn to like to do things which we are convinced may be of advantage to us. We are reminded of Benjamin Franklin, who says of himself when about sixteen: "And now it was that, being on some occasion made ashamed of my ignorance in figures, which I had twice failed in learning when at school, I took Cocker's book of Arithmetick, and went through the whole by myself with great ease." When pupils are convinced that mathemathics is worthwhile to them they have a proper motive for work and they will rejoice in the achievements which it makes possible.

Mathematics as an elective study in high schools may be made more secure than it was when required of all. When its importance is properly understood by all, it will be elected eagerly, and by all. To gain this will require a more active interest on the part of both the mathematician and the teacher. The teacher can work most effectively through the interest he creates in the class room, but he should use every legitimate means to make effective that which he has elected as his life work.

A MATHEMATICS CLUB

By MARY CAROLINE HATTON
Graduate Student, Teachers College

In November, 1923, the mathematics department of Leonia High School, Leonia, N. J., realizing the wealth of material in mathematics that cannot be touched in the classroom because of lack of time, organized a club to supplement its classroom work. Any student of Grades X, XI, or XII who is enrolled in a mathematics class is eligible for membership in the club, but the constitution which the students have drawn up, states that no officer may be chosen from the lowest class, and the honor of presidency may be bestowed only upon a Senior. The meetings are held semi-monthly, at the close of the last recitation period of the day. The projects and programs of the club are carefully worked out by committees of students under faculty supervision. These activities are planned to meet the definite aims of the organization. Each student in the club is well acquainted with the fact that these aims are:

I. To open up possibilities in the field of mathematics for a worthy use of leisure time.

II. To create an interest in the practical application of mathematics as the specialist uses it to-day.

III. To acquaint the student with the scholarly work that has been done in the development of mathematics, and to stimulate admiration for the scientists in this field and their work.

The club opened its activities with a slide rule project. The slide rule had never been used in the high school before, and was a source of wonder at first. The club had in its treasury a little money which had been collected from the members for dues. With this money a supply of inexpensive rules was purchased. It was not long before many of the students bought better rules of their own. Contests of speed and accuracy were held at every meeting, and it certainly took no skill on the part of the faculty advisers to stir up interest. Every day at lunch time and after school one could have seen a group of the club members "sliding" away with their rules, surrounded by a group of admiring and curious non-members, eager to solve the

mystery that lies hidden in the magic stick. The students of Trigonometry made the greatest immediate use of the slide rule in their class room work, but it was also used to a limited extent in the other mathematics classes.

As soon as the weather permitted, instructions in the use and manipulation of a transit, and the carrying out of five surveying problems received the enthusiastic support of the whole club. Several faculty members were kind enough to loan transits to the club. The work was done by groups of students. All of the students recorded the results of the field work in note books on which a great deal of time and effort was spent. It might be added that to improve the appearance of the books the art teacher gave the club a very good lesson in the making of engineers' figures. Following is a list of the work done in the field:

(1) To find the difference in elevation of points on a level circuit.

(2) To find the height of an object with an accessible base.

(3) To find the height of an object with an inaccessible base.

(4) To run an open traverse.

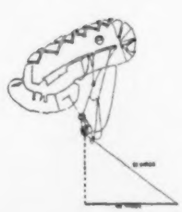
(5) To run a closed traverse.

In connection with the surveying work the club was very fortunate in having for its guest Mr. John E. Dixon, Eagle Scoutmaster at Perth Amboy, N. J. Mr. Dixon gave a most interesting talk on the art of surveying, showing how some of the problems the club had just done in the field could have been done without the use of instruments. He also described in detail the early surveying instruments. He showed how the cross-staff, the square and the quadrant were used. After his talk a group of his scouts illustrated the methods of measurement as set forth in the Boy Scout Manual. Several of the boys in the club became so keenly interested in the early instruments that they decided to make some themselves. As a result the club has two very well made quadrants. The boys explained the use of them to the other members, and then a group used them to measure the height of the same pole that had been previously measured by means of the transit. The results were found to compare favorably with the former data. All of this work fitted admirably, of course, with the teaching of similar triangles in Geometry, and with the solution of triangles in Trigonometry.

A Mathematics Week was celebrated in the high school under the club's auspices toward the end of the first year of its existence. In each of the mathematics classrooms exhibitions of classroom work were held. Note-books, projects and geometrical designs were displayed. A contest was held between the various sections of Plane Geometry students. First of all a series of tryouts was given in each section and the eight students who gained the highest number of points were given the honor of representing their respective sections in the final contest. This contest was divided into three parts. The first part was a series of twenty-five completion sentences very similar to the sentences in Part 1 of the Schorling-Sanford Achievement Test in Plane Geometry. The second part consisted of the formal oral proof of five propositions. A faculty adviser put one figure on the board at a time and stated the theorem to be proved. Each team took turns in letting its members in order give on statement or reason in the logical development of the proof of the theorem. If any member failed to give the correct statement it brought a mark against his team. The third part of the contest consisted in the solution of five original problems. The teams were sent to different parts of the room, each team facing a section of blackboard, provision being made that no team could easily see the blackboard space alotted to any other team. The same figure was put on the board space belonging to each team, and then the problem was read. The teams wrote out the solution of the problem in their allotted space, one student at a time going to the board and writing a statement and reason needed in the proof. No student was allowed to write two successive statements. Beyond that there were no restrictions concerning which members of the team should do the solving. Absolute silence was required of the teams during the contest. A definite outline of scoring had been made out by faculty members previous to the contest. These same faculty members acted as umpires throughout the game. No athletic contest in the school ever aroused more excitement and interest than that matching of wits in mathematics. The teams certainly had the loyal support of their sections, every student fully aware that honor was due to those energetic contestants.

As a special treat of the week some of the members of the club presented the little farce "The Mock Trial of A vs. B" in the auditorium of the school during an assembly period. If the applause, laughter and comments may be regarded as criteria the play was assuredly a success. Indeed the whole week was a pleasure, and the great spur it gave to the mathematics classes could not possibly go unnoticed by a teacher of that department of the school. The club had its final meeting of that year with the consciousness that it had done a worthy service for the school.

The opening meetings of the second year of the club were again spent with the transit and slide rule. The new members were given at least an idea of what had been done. In November that part of the year's work was ended by a big slide rule contest, and the new project was introduced. This was a series of meetings dealing with the games, puzzles and fallacies that can give one so much entertainment. The students brought in many interesting problems that they found in current magazines. At one meeting the scarcity of interesting and practical problems in mathematics textbooks was discussed and followed by an attempt on the part of the students to formulate some which they considered worth while. Attached is the work done that day by one youth whose idea of a real day of sport



would evidently be one which brought him a balloon ride and a fishing trip. One of the boys asked for the assistance of the club in helping him meet a difficulty with which he had been confronted at the garage where he was working after school hours. The garage had its cylindrical gasoline storage tank lying lengthwise under ground. The length and diameter of the tank were known, but there was no gauge for computing how much gasoline the tank contained at any given time. The owner of the

garage suspected that the people with whom he was dealing were not giving him his full order of gasoline, but he had no way of checking it up. The boys in the club marked off on a stick a distance equal to the diameter of the tank. This they divided into spaces two inches in length. When the stick was inserted through the opening on the side of the tank, and lowered to the opposite side, of course, the depth of gasoline left its mark, and they computed the contents with each of the division marks registered. It was merely the volume of a lengthwise cross-section of a cylinder, but it was the first time any of the members of the club had ever tried to find it, and they managed their problem very well indeed. This type of work led to a study of the sources and development of the most familiar problems that have come down to us. The problem of the pipes filling the cistern with its many variations, the one of the Turks and Christians, the testament problem, and problems of pursuit, were all discussed. This work was thoroughly enjoyed, and the fount of Mathematics found to be an unlimited source of pleasure.

In the midst of this work, the club and its friends had the rare opportunity of hearing Professor Yalden of the American Astronomical Society explain the cause of the coming eclipse of the sun. He also gave very definite instruction on the methods of observing the eclipse, and asked for the co-operation of the club in obtaining pictures and data on the morning of the eclipse in this particular section of New Jersey.

The next project entered upon by the club was a historical one. The work with the problems had aroused interest in history and therefore made the subject easy to develop. The development of Arithmetica and Geometrica, mechanical aids to calculation, the three famous problems in Geometry, the history of the solution of equations in Algebra, and the origin of our units of weights and measures were all studied and interestingly presented at the club meetings. "History of Mathematics," Vol. II, by David E. Smith, gave valuable material in developing this phase of the work.

Looking ahead to future activities the club felt that it ought to take advantage of some of the films that can be produced which give reliable information on mathematics and subjects

closely allied to it. Some of these films may be obtained free of charge. Others are rather expensive. To make the film project possible for next year, the club obtained permission from the principal of the school to give a motion picture show in the auditorium after school one Friday. The boys attended to the selection of the film and the running of the machine; the girls took charge of the advertising and tickets. They worked so hard convincing their fellow students that they should patronize the "movie," that the treasury of the club now boasts almost $50.00 which will be well used for the renting of educational films next season. Probably the first picture that will be shown will be "The Benefactor," a three-reel motion picture of the life of Thomas A. Edison, which can be obtained without charge from the Publication Bureau, General Electric Co., Schenectady, N. Y. The General Electric Co. has some other films of mathematical interest which are well described in their pamphlet "Lecture Service, Motion Picture Films, Lantern Slides." This pamphlet may be obtained at the above address. The Argonaut Distributing Corporation, 5 Columbus Circle, New York City, has some interesting films to be used in connection with mathematics. The New Jersey State Department of Public Instruction will send out its films to clubs applying for them. Mr. Chas. H. Sampson of the Massachusetts Department of Education has completed a film entitled "Definition of Plane Geometry." However, any club wishing to develop a screen project should first obtain a copy of "1000 and One" from the Society for Visual Education, Inc., 130 West 46th St., New York City. This lists the best films that can be procured. In it are listed seventeen films under "Engineering Achievements," all of which would be suitable material for Mathematics Club work. Two of these that the Leonia Club plans to show are "Construction of Subway Tubes," a picture featuring the building of New York's tubes under the Harlem River and "Repairing a Subsea Cable." This latter film shows the raising of a transatlantic cable for examination and repair. "1000 and One" also introduces twenty films on elementary astronomy. Of these the Leonia Club expects to use "The Earth and Worlds Beyond" in the near future. The same pamphlet lists twenty-one films on military and naval subjects from which number several of mathematical

nature may be chosen. The offerings in the field of physics are especially good in film-land. Some of these will, of course, fit in admirably for the Mathematics Club. For example "Einstein's Theory of Relativity," a simple and exceedingly instructive picturization of Einstein's theory is an ideal piece of material. These few pictures that have been mentioned are merely suggestions taken from the great bulk of material that is availabl for projection.

The Leonia Mathematics Club has done much to vitalize the mathematics classes and has succeeded in making itself felt throughout the school. It has had a successful past, and its members are hopefully looking forward to its growth and higher achievements in the future.

ANALYSIS VERSUS SYNTHESIS

ALMA M. WUEST
Walnut Hills High School
Cincinnati, Ohio

One of the major issues in the teaching of demonstrative geometry is to determine the relative values of the analytic and synthetic methods of proof. The usual method employed is the latter and although it is in the newer books that we find more use made of the analytic method, the analytic method is not new. Plato is said to have invented this form of proof. To estimate the relative values of these two methods let us examine them in the light of the aims for teaching demonstrative geometry. If we can decide why a pupil should study geometry then it will be comparatively easy to determine which of these two forms of proof will help us achieve the desired results.

Geometric facts are not useful in every-day life, so we can not defend the teaching of demonstrative geometry on the basis that it gives the pupil a quantity of useful knowledge. It is true, however, that geometry does teach the pupil how to think, how to reason, how to analyze, how to discover new facts and only on this basis can we justify the retention of demonstrative geometry as a subject in our high school course. Hence the principal aim of teaching geometry is to give the pupil power to think out the solutions of problems and not to give him a knowledge of geometric facts. Since we are striving to give the pupil power to analyze new situations rather than a store of geometric propositions we must stress that phase of demonstrative geometry which develops analytical thinking rather than encourage the memorizing of synthetic proofs.

Analysis then becomes one of the essential features of geometry teaching. It is the process by means of which the pupil assumes what he wants to prove as being true, sees what results follow step by step, until one of these steps connects with the known. In this way he has made a path from the unknown to the known by tearing apart the original statement into simpler

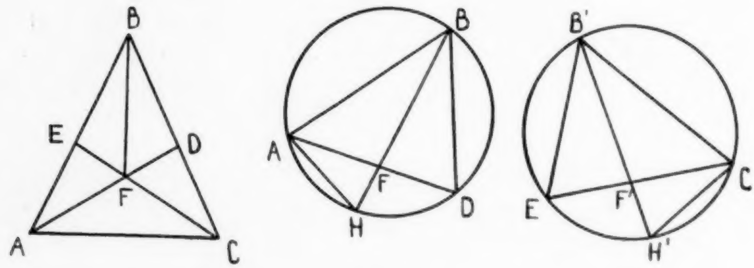
statements and establishing the truth or falsity of these simpler truths. Every thinker, whether he be a business magnate, a scientific investigator, a tenth-grade student or a mathematical worker, discovers and establishes new truths in this way. Consequently we should find analytical thinking everywhere in the school and where can it be used to a better advantage than in the demonstrative geometry class?

Can we call this analysis, as used in geometry, the raising of questions and discussing various possibilities step by step until we reach the step that connects with the known, a logical proof? If by logical proof you mean an irrefutable argument, and if the steps in the analysis are each supported by a legitimate authority, then it is a proof. It is an informal proof that not only establishes the truth but also helps the pupil discover the proof. This type of training will prepare the pupil to attack propositions himself and work out their solutions, thus giving him the desired training in thinking which will enable him to work out successfully problems in other school subjects or in life.

The synthetic method, too, has a place in the class room. By this method the pupil is able to put together the known truths and arrive at the unknown truths. After a pupil has worked out the problem of analysis, then he should use the synthetic method to arrange his discoveries for future reference, permanent records, reviews or formal proofs. The synthetic proof is formal, concise and finished, and thus is economical from the standpoint of time when it comes to writing down the proofs. This method should never be used for instruction. If it is used, the pupil will be able to follow the reasoning step by step but he is not able to see why the steps that were taken were taken. He follows the reasoning blindly, he is convinced that the argument is correct but he has not gained any intellectual growth from the process, no power to attack more difficult problems later in the course.

An example of an analytical discovery and proof can be found in the solution of the following exercise.

Ex: If two angle bisectors of a triangle are equal, the triangle is isosceles.



Given: $\triangle ABC$ in which AD and EC bisect $\angle A$ and $\angle C$ respectively, and $AD = EC$.

To Prove: $AB = BC$

It is easily seen that if $AB = BC$, then $\triangle ABD = EBC$ (s.a.s.). Can it be proved that $\triangle ABD = EBC$?

They have

$$AD = EC \text{ (hyp.)}$$
$$\angle ABD = \angle EBC \text{ (common)}$$

these two facts are not enough to prove two triangles equal. If you draw BF, the bisector of $\angle B$, you will notice that the two triangles have the bisectors of their equal angles equal.

The problem now has reduced itself to this: Are two triangles equal if they have one side, the opposite angle and its bisector, of one triangle equal respectively to one side, the opposite angle and its bisector, of the other? Draw two triangles as $\triangle$'s ABD and $EB'C$ which fulfill these conditions.

Given: $\triangle$'s ABD and $EB'C$, with $AD = EC$, $\angle ABD = \angle EB'C. BF$ bisecting $\angle ABD$ and $B'F'$ bisecting $\angle EB'C$ and $BF = B'F'$

To Prove: $\triangle ABD = \triangle EB'C$

The fact that the equal angles were opposite the equal sides suggests the construction of a circular segment on the given

line as a base that shall contain the equal angle. Describe a circle about each triangle.

> (Only one segment of a circle can be constructed on a given line, $AD=EC$, that shall contain a given angle, $\angle ABD=\angle EBC$)

If $\triangle ABD = \triangle EB'C$, then $\angle D = \angle E$ and $AB = B'C$. We know $\overset{\frown}{AD} = \overset{\frown}{EC}$ ($AD = EC$ by hyp.) If you produce BF and $B'F'$, the arcs $\overset{\frown}{AD}$ and $\overset{\frown}{EC}$ are bisected, $\therefore \overset{\frown}{BAH} = \overset{\frown}{B'CH'}$, and $BH = BH$. Since we know that $BF = B'F'$ we must discover a way to prove $FH = F'H'$. This suggests triangles, so draw AH and $H'C$. It is impossible to prove these triangles, $\triangle AFH$ and $\triangle F'H'C$ congruent. The idea then of similar triangles was taken up to see if we could get any assistance from proportions.

$$\triangle AHF \sim \triangle AHB, \text{ hence } \frac{BH}{AH} = \frac{AH}{FH}$$

$$\triangle H'F'C' \sim \triangle H'B'C', \text{ hence } \frac{B'H'}{H'C'} = \frac{H'C'}{F'H'}$$

Comparing these two proportions and noting that $AH = H'C$, $B'F' = BF$, $BH = BF + FH$ and $B'H' = B'F' + F'H$ we get $FH = F'H'$.

This final step connects with the known fact $BF = B'F'$ and hence the problem is solved.

NOTE: See Schulze and Seven Oak's *Plane Geometry* (Revised) Macmillan Co., 1913; Exercises 1385, 1384, 1383, 1061.

NEW BOOKS

Teaching Tests in Algebra.—In no other subject in the field of secondary school mathematics where educational reforms of the day are beginning to show influence are there more pronounced results than in the teaching of algebra. Due to the criticisms from all sides, the teachers of the subject have had to organize their efforts in order to justify the place it should occupy in the curriculum. The college professor has claimed that the pupil came to him with faulty preparation; the science teacher has claimed that the pupil knew no mathematics; the secondary school educator was looking for a better product; the supervisor has claimed that it was the method of instruction; and the general public wanted evidence of a product which functioned in the everyday activities of life. As a consequence, the leaders have reorganized the subject matter and modified their operative techniques. In the first place, many new textbooks have appeared, bearing the claim that they were strictly in accordance with the reform movement. In the second place, many new teaching devices have been propounded. One of these devices is known as the teaching test. Its object is the application of certain definite principles underlying the psychology of learning. One of the main phases is to have the pupil become interested in his own progress. Improvement in achievement increases most rapidly when the individual begins to observe his own learning methods and becomes the appraiser of his own progress. In this connection the evaluation of one's own power and skill will be far more effective if it is based on well defined psychological and pedagogical principles. It is a well-known fact that many authors have not duly observed these principles in the field of algebra. It then remains for the teacher to seek relief through the aids which are available.

A very recent publication of this type is a little booklet, "*Instructional Tests* in *Algebra.*"[1] The authors place great emphasis upon the fundamental principles of the psychology of drill and

---

[1] Schorling, Raleigh, Clark, John R., and Lindell, Selma A. "*Instructional Tests in Algebra.*" Chicago: World Book Company, 1927, 72 p.

urge the teachers of algebra to apply these principles in their regular classroom activities. They believe that this booklet in the form of 52 instructional tests makes it possible to utilize these principles.

These tests are to be used as supplementary material to the ordinary algebra text. They are so constructed that they will fit any elementary course, and the organization is such that the pupil will realize that the important tasks of algebra are real goals to be attained.

The technique is such as to make the administration of the tests very simple. The steps are: (1) The tests should be given at the beginning of the recitation; (2) Start and stop all pupils at the same time, allow exactly 8 minutes; (3) Have the pupils score their work by referring to the answers in the back of the booklet. The two important factors involved are the timing of the tests and the recording of the results. Standards for each test are suggested at each of three levels of ability, having been determined by careful study and trial under varied classroom situations. Instructions are given to the pupil that a test is passed when one of the levels is reached, but that he need not be satisfied until the highest standard is attained.

This is a method of determining the final measure of the pupil's skill and can be utilized for diagnosing individual weaknesses. In other words, it is an attempt to adapt procedure to individual differences. The authors say to the pupil: "You will be allowed to travel at your own rate."

Another recent booklet introducing the use of teaching tests is, "*Exercises and Tests in Algebra.*"[2] In this publication the authors emphasize "No skill without drill" and "Practice makes proficiency." Such a procedure is based on sound psychology. In this booklet there are abundant drill and test material which may be adapted to any elementary course in algebra. The authors have developed these tests from classroom situations and through their repeated applications the accurate performance of the essential operations of algebra should become habitual. It is a platitude to say that the success

---

[2] Smith, David Eugene, Reeve, William D., and Morss, Edward L. "*Exercises and Tests in Algebra.*" Boston: Ginn and Company, 1926, 224 p.

of later work in algebra, science, and their everyday applications, depends upon skill in these essential operations. In the present instance, great emphasis is placed on the topics which are useful in science and practical work.

With a suitable teaching technique, these tests are complete and diagnostic, testing more or less the specific abilities which are involved in the learning of algebra. Thus, specific weaknesses are revealed and the opportunity for remedial work is offered. The two forms, the odd-numbered and the even-numbered tests which are comparable in a large degree, make the remedial feature of algebra teaching an interesting and effective one.

There are 224 tests in all and they include all the topics taught in the usual elementary course—plus quadratics. The simple instructions which precede the tests make the administration a simple matter. Each test has a time limit and thereby may be made a valuable measuring instrument in the pupil's hands for the purpose of evaluating his own progress.

This is a method of taking inventory. Instead of sampling the pupil's knowledges and skills by the traditional examination and recitation method, a means is available for a more searching diagnosis of achievement. It is a means of correcting the various factors causing maladjustment to the differences existing among individual pupils.

C. N. Stokes,
University High School, University of Minnesota.

PROGRAM FOR THE SEVENTH ANNUAL MEETING

OF THE

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

AT DALLAS, TEXAS,

February 25 and 26, 1927.

FRIDAY EVENING SESSION, 7:30 P. M.
At the Hilton Hotel

Joint meeting of the Executive and Local Committees.

Important items of business will be discussed at this meeting. Every member of each committee is urged to attend.

SATURDAY MORNING SESSION, 9:00 A. M.
At the Adolphus Hotel,
Bambooland, West.

BUSINESS MEETING.

Report of the Secretary
Report of the Treasurer
Report of the Auditing Committee
Report of the Editor
Report of the President
New Business

REPORTS OF CLASSROOM EXPERIMENTS (20 minutes each)

Investigations in the Teaching of Plane Geometry
John R. Clark, The Lincoln School, New York City

The Laboratory Method in Teaching Geometry
William A. Austin, Head of Mathematics Department, Venice, California.

Individual versus Group Instruction in Ninth Grade Algebra
C. B. Marquand, West High School, Columbus, Ohio.

Efficiency of Instruction in Large and Small Classes
Leonard D. Haertter, John Burroughs School, Clayton, Missouri.

DISCUSSIONS (5 minutes each)

SATURDAY NOON, 12:30 P. M.
At the Adolphus Hotel
Luncheon for the Executive and Local Committees

SATURDAY AFTERNOON SESSION, 2:00 P. M.
At the Adolphus Hotel
Bambooland West

How to Make the Concept of the Locus Real
Elsie Parker Johnson, Oak Park High School, Oak Park, Illinois.

The Romance of the Number System
H. E. Slaught, University of Chicago, Chicago, Illinois.

Some of Euclid's Algebra
George W. Evans, Houston, Texas.

SATURDAY EVENING SESSION
At the Adolphus Hotel
Palm Garden

Banquet, 6:00 P. M.

Members are requested to send their reservations to Miss Elizabeth Dice, North Dallas High School, Dallas, Texas. The price of the banquet is two dollars and fifty cents a plate.

GREETINGS FROM GUESTS OF HONOR

PRESENTATION OF THE SECOND YEARBOOK
W. D. Reeve, Columbia University, New York City.

DISCUSSION

NEWS NOTES

At the Buffalo meeting of the Association of Teachers of Mathematics in the Middle States and Maryland, held November 27, 1926, the following officers were elected: President, Prof. Wilfred H. Sherk, University of Buffalo; Vice-President, Prof. E. E. Rich, Lawrenceville School, Lawrenceville, N. J.; Secretary, Prof. W. D. Reeve, Columbia University, N. Y.; Treasurer, E. O. Bull, State Normal School, West Chester, Pa.

Professor Raleigh Schorling of the University of Michigan spent the month of December in California, lecturing on the teaching of secondary mathematics.

The program of the Topeka Mathematics Round Table included: I. The New Arithmetic—Miss Carmille Holly, K. S. T. C., Emporia; II. Minimum Essentials and Methods in High School Algebra. 1. Symbolism and Formula, Miss Zella Colvin, Oread Training School, Lawrence; 2. Directed Numbers, Miss Nina McLatchey, Topeka High School, Topeka; 3. Factoring, Prof. U. G. Mitchell, K. U., Lawrence; 4. Equations, Mr. Delbert Emery, Manhattan High School; 5. Fractions, Miss Lucy T. Dougherty, Junior College, K. C., Kansas; 6. Order of Topics, Mary E. Helwig, High School, Kansas City, Kansas; 7. Discussion and questions. III. The Relation of the Round Tables to the Kansas Association of Mathematics Teachers, Miss Emma Hyde, K. S. A. C., Manhattan. IV. Business Meeting: 1. Consideration of Publication of a Bulletin; 2. Mathematical Literature, Miss Ethel Rumney, K. S. T. C., Emporia; 3. The State Texts, Miss Edna Austin, Topeka High School; 4. Election of Officers.

The Mathematics Round Tables in the four sections of the Kansas State Teachers' Association followed a new plan for their meetings this fall which proved very satisfactory. The programs given were essentially the same; the principal numbers in each being "The New Arithmetic"; "A Symposium";

"Minimum Essentials and Methods in High School Algebra"; and "The Relation of the Mathematics Round Tables to The Kansas Association of Mathematics Teachers."

The outstanding thought of the program was that of mutual helpfulness. Through bringing the Round Tables and the Association into closer contact, the Association will reach fully eight times as many teachers and hence be of far greater service in its efforts to improve the mathematics teaching in our high schools. One suggestion which may prove valuable in bringing this about was that of the publication, by the Association and the Round Tables, of an eight-page mathematics bulletin, to be sent to each member of the Association whose dues are paid. The suggested contents of the bulletin to be about as follows: A statement of the issues decided upon; how they are to be carried out; suggestions on how to present difficult parts of the work for new teachers; the progress of the work on the new course of study; discussions of textbooks and how to use them; general mathematical news; possibly a question page; and personal news of mathematics teachers. The plan was initiated by Miss Ina E. Holroyd of the Department of Mathematics of the Kansas State Agricultural College; and a four-page sample bulletin prepared by her was distributed at each of the Round Tables.

At the Topeka meeting, which was one of the best in its history, a discussion of the State Algebra Text in the senior high school was led by Miss Edna Austin of the Topeka High School. The Round Table instructed its chairman to convey to the State Textbook Commission its willingness to cooperate in the selection of a suitable text; and further to authorize the Kansas Association of Mathematics Teachers to act as its representative in such action in the matter as seemed advisable.

The chairmen of the four Round Tables were as follows: Chanute, Mr. J. H. Koontz of the Parsons High School; Hutchinson, Mr. W. E. Bereman of the Nickerson High School; Topeka, Miss Ina E. Holroyd of the Kansas State Agricultural College, Manhattan; and Hays, Miss Anna Marm of Bethany College, Lindsborg.—(Contributed by Miss Ina E. Holroyd of Kansas Agricultural College.)

The Kansas State Association of Mathematics Teachers, through its president, W. D. Stratton, and a large committee of the Association, has published a mathematics bulletin in which the activities of the Association are described. The introduction of the first number of the bulletin follows:

WELCOME NEW AND OLD!

Let us rally to the slogan: *Every Mathematics Teacher in Kansas a member of the Kansas Association of Mathematics Teachers!* We welcome to our ranks the new members, and bespeak the hearty support of the old members for the year 1926-27. Let us unite to make Mathematics the best, the most practical, the most inspiring of all the subjects taught—an honor to our profession and a force which will function in the lives of Kansas boys and girls.

Self Examination of the Mathematics Teacher

1. Do I have an aim?
2. Do I plan my lessons in advance?
3. Do I satisfy myself with having discovered something to keep the pupil busy during the recitation period?
4. Do I know the individual needs of my pupils?
5. Do my pupils get the maximum benefit from the work I ask them to do?
6. Do my pupils get a training in how to think in general terms which will carry on?
7. Do I inspire my pupils with a desire to know more about my subject?
8. Do my pupils realize that mathematics is an absolute necessity as a prerequisite to the study of any science?
9. Without neglecting the mediocre, do I make the most of my "bright pupils?"
10. Do I demand good English in my class-room?

Q. How can I improve myself as a teacher of Mathematics?

A. By becoming an active member of the Kansas Association of Mathematics Teachers.

The eleventh annual meeting of the Mathematical Association of America was held in Philadelphia December 30-31, 1926. The program included:

1. "A Mathematical Critique of Some Physical Theories," Professor G. D. Birkhoff, Harvard University, Retiring President of the American Mathematical Society.

2. "The Weight Field of Force of the Earth," Professor W. H. Roever, Washington University, Retiring Vice-President of Section A of the American Association.

3. "The Duty of Exposition with Special Reference to the Cauchy-Haveside Theorem," Professor F. D. Murnaghan, Johns Hopkins University, Representing the Mathematical Association.

4. "Cauchy's Integral Theorem," Professor D. R. Curtiss, Northwestern University.

5. "On Various Conceptions of Correlation," Professor F. M. Weida, Lehigh University.

6. "Here and There in Europe," Professor R. C. Archibald, Brown University.

7. "The Use of the Trinometer in Engineering and in the Teaching of Mathematics," (exhibiting an instrument), Professor J. E. Rowe, College of William and Mary.

8. Annual Business Meeting and Election of Officers.

9. "Two Geodesists of the Eighteenth Century," Mr. W. D. Lambert, U. S. Coast and Geodetic Survey.

10. "Modern Methods and Results of Stellar Parallax Investigations," Professor J. H. Pitman, Swarthmore College, by invitation. Illustrated by lantern slides.

11. "The Maintenance of Mathematical Interest," Professor E. R. Hedrick, University of California, Southern Branch.

12. "The Apportionment of Representatives in Congress," Professor E. V. Huntington, Harvard University.

The program was prepared by Professors: A. A. Bennett, Lehigh University, Chairman; H. B. Evans, University of Pennsylvania; Tomlinson Fort, Hunter College; Florence P. Lewis, Goucher College; E. P. Morrow, Gilman Country School.

The program of the December meeting of the Association of Mathematics Teachers in New England included: (1) A Number of "Things" for Beginners in Geometry, by Vesta Rich-

mond, High School, Newton, Mass.; (2) Mathematics and Culture, by Professor Marston Morse, Harvard University; (3) Mathematics in a Special High School, by Harold B. Garland, High School of Commerce, Boston; and (4) Alternatives to Ruler and Compass Construction, by Professor Phillip Franklin, Massachusetts Institute of Technology.

Miss Carmilla Holley of the Kansas State Teachers College at Emporia, Kansas, and Professor W. T. Stratton, President of the Kansas State Teachers Association obtained 43 memberships in the National Council of Teachers of Mathematics at the November meeting of the Association.

Mr. C. M. Austin of Oak Park, Illinois, has obtained more than 40 memberships in the National Council during the last month.

Dean J. H. Minnick, School of Education, University of Pennsylvania, is editor of a new magazine, "Educational Outlook," which is published by the School of Education of the University of Pennsylvania. Dr. Minnick is known for his contributions to the teaching of mathematics, particularly for his researches in the analysis of the abilities in geometry.

THE NATIONAL COUNCIL IN LOUISIANA-MISSISSIPPI

By PROFESSOR H. E. SLAUGHT
University of Chicago

A concerted effort is in progress to establish a branch of the National Council in Louisiana-Mississippi. The sponsors of this program are the officers of the Louisiana-Mississippi Section of the Mathematical Association of America. The annual meeting of this Section, which usually comes at the end of March, has been fixed for March 4-5, 1927, in order to secure the attendance of officers and other members of the National Council who will be returning from the Dallas meeting of the Council which occurs the previous week.

The Louisiana-Mississippi meeting will be held at Shreveport, Louisiana, which is about 200 miles directly East of Dallas and can be easily reached on the return route.

The Mathematical Association of America, which sponsored the National Committee on Mathematical Requirements, is now deeply interested in spreading the influence of the National Council. The Louisiana-Mississippi Section of the Association is urging all secondary teachers of mathematics in these states to attend the Shreveport meeting and to organize a branch of the National Council to cooperate with the Association and to hold meetings in conjunction with it to the end that the college and secondary teachers may study together their mutual problems and thus strengthen the entire mathematical situation in these states.

The Association is urging the secondary teachers to join the National Council and to become readers of the Mathematics Teacher. For any secondary teachers who may wish to join the Association, the initiation fee of two dollars will be waived in case they are already members of the Council. All the high schools in these states are being urged to send at least one delegate with expenses paid, to the Shreveport meeting. If only one-half of the schools respond favorably, it will make a notable gathering. All members of the Council who attend the Dallas meeting are urged to stay over for the Shreveport meeting.

MEMBERS OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

(Continued from December issue)

PENNSYLVANIA

Katherine R. Fowler, 668 Baldwin St., Meadville.
Marguerite McClintock, 473 Walnut St, Meadville.
Louise F. McClellan, 229 E. Butler St., Mercer.
Henry J. Hollinger, Myerstown.
Vera Frost, 514-9th St., Monessen.
Amelia Richardson, Lincoln St., Monongahela City.
John C. Bechtel, 103 W. Nippon St., Mt. Airy.
Blanche Galley, 103 Church St., Mt. Pleasant.
Supt. C. R. Stone, Munhall Public Schools, Munhall.
Wm. P. Nash, Ardmore.
Roosevelt Junior High School, c|o J. Fred McMurray, 2229 West 4th St., Nerberry.
Raymond Schlosser, Bonair and Eagle Rd., R. F. D. No. 3, Newton Square.
H. B. Banhower, 1032 N. Marshall St., Norristown.
Winfield R. Hartzell, 718 Stanbridge St., Norristown.
Maylew Stritzinger, 520 Hamilton St., Norristown.
Agnes E. Thomas, Ogontz School, P. O.
Ethel M. Hicks, Reiffton, Berks County.
J. B. Derr, Supervising Principal, Perkasie.
Dr. Jacob B. Krause, 3037 N. Broad St., Philadelphia.
Cooke Junior High School, 13th & London Sts., Philadelphia.
Jones Junior High School, Ann & Tulip Sts., Philadelphia.
Roosevelt Junior High School, E. Washington Lane & Musgrave St., Philadelphia.
Tilden Junior High School, 70th St. & Buist Ave., Philadelphia.
May McKelvey, Box 232, New Wilmington.
Roxborough Junior High School, Ridge Ave. & Fountain St., Philadelphia.
Amy Lilley Clapp, 3809 Spruce St., Philadelphia.
Mary L. Constable, 1827 Mt. Vernon St., Philadelphia.
Miss J. Burnette Hallowell, 3417 Baring St., Philadelphia.
Alice M. Holbrook, High School for Girls, Spring Garden & 17th Sts., Philadelphia.
Sam Katz, 3316 N Bouvier St., Philadelphia.
Mrs. Adele U. Keller, Kensington High School, Carol and Cumberland Sts., Philadelphia.
Margaret Kerr, 4910 Wynnefield Ave., Philadelphia.
Elizabeth S. Lachey, 2525 N. 5th St., Philadelphia.
Agnes H. Long, Wm. Penn High School, 15th and Mt. Vernon Sts., Philadelphia.
Lillian Moore, 885 N. 28th St., Philadelphia.
Mr. Lee J. Pennepacker, 867 Preston St., Philadelphia.
C. L. S. Raby, 4616 Pelling St., Frankford, Philadelphia.
Joseph Riccinti, 1531 S. 10th St., Philadelphia.
Jonathan T. Rorer, 15th and Mt. Vernon Sts., Philadelphia.
Anna Sensenig, 778 N 41st St., Philadelphia.

Harding Junior High School, Forresdale Ave. and Wakeling St., Philadelphia.
Holmes Junior High School, 55th and Chestnut Sts., Philadelphia.
Thomas Junior High School, 9th and Johnston Sts., Philadelphia.
Mrs. Elizabeth Roesing, 2336 E. Dauphin St., Philadelphia.
Mabel V. Jaquette, 5904 Washington Ave., Philadelphia.
Stetson Junior High School, B St. and Alleghany Ave., Philadelphia.
South Philadelphia High School for Boys, Broad and Jackson Sts., Philadelphia.
Pedagogical Library, 2nd floor Keystone Bldg., 19th St. above Chestnut, Philadelphia.
Harry M. Shoemaker, 1906 N. 13th St., Philadelphia.
Helen O. Shollenberger, 4511 Pine St., Philadelphia.
Muriel Smith, West Phila. H. S. for Girls, 47th and Walnut Sts., Philadelphia.
Margarel Sproul, 1539 N. Gratz St., Philadelphia.
Esther M. Venables, 220 S. 43rd St., Philadelphia.
Marie Wilhelme, 15th and Mt. Vernon Sts., Philadelphia.
Nettie Wurster, 147 W. Hortter, Germantown, Philadelphia.
Elizabeth H. Wood, 4631 Sansom St., Philadelphia.
Florence Yonkers Bishop, Frankford High School, Frankford, Philadelphia.
E. B. Ziegler, 5119 Cedar Ave., Philadelphia.
Harry B. Marple, 6017 Webster St., Philadelphia.
Eleanor K. Cooper, 4503 Kingsessing Ave., Philadelphia.
Louis Gordon, 4720 N. 7th St., Philadelphia.
Room 307 High School Bldg., Girard College, Philadelphia.
Minnie Bernstein, 823 S. 59th St., Philadelphia.
Julia M. Bligh, 4229 Regent St., Philadelphia.
E. H. Carl, 6418 N. Park Ave., Philadelphia.
Edna E. Cassel, 4500 N. Broad St., Philadelphia.
Grace Christian, 2725 East Lehigh Ave., Philadelphia.
Vare Junior High School, 24th St. and Snyder Ave., Philadelphia.
Donald E. MacCormick, Wm. Penn Charter School, West School, Lane and West, Wissohickon Ave., Germantown, Philadelphia.
School for Blind, 64th & Malvern Ave., Overbrook, Philadelphia.
Ella Rosentoor, 5300 Lebanon Ave., Philadelphia.
Miss M. A. Jones, Friends' Select School, The Parkway and 17th St., Philadelphia.
Margaret Groff, S. Phila. High School for Girls, Broad St. and Snyder Ave., Philadelphia.
Johanna L. Jaeger, 3312 Benner St., Philadelphia.
Mr. E. Anders, 5312 N. Camac St., Philadelphia.
Lawrence M. Babcock, 1711-67th Ave., Oak Lane, Philadelphia.
William Boerchel, 142 E. Wyoming Ave., Philadelphia.
Miss G. C. Ellis, 1538 Master St., Philadelphia.
Charles Friedman, 2713 S. 7th St., Philadelphia.
Miss H. E. Mann, 2022 N. 18th St., Philadelphia.
Ethel M. McAllister, 5112 Chester Ave., Philadelphia.
Dr. J. H. Minnick, School of Education, University of Pennsylvania, Philadelphia.
Henry S. Moore, 718 E. Price St., Philadelphia.
Anna M. Breadin, 3041 Susquehanna Ave., Philadelphia.
David M. Hass, Central High School, Broad and Green Sts., Philadelphia.
Elmer A. Jacoby, 7113 Boyer St., Philadelphia.
William I. Jones, 1222 Rising Sun Ave., Philadelphia.

Helen Lamberton, 4503 Kingsessing Ave., Philadelphia.
Margaret Ligon, 1612 N. 15th St., Philadelphia.
Walter Roberts, 3419 N. 22nd St., Philadelphia.
Florence Rothermel, 320 S. 42nd St., Philadelphia.
James B. Summers, 4939 Larchwood Ave., Philadelphia.
Evelyn F. Alden, 1838 Wallace St., Philadelphia.
Grace L. Bentley, 1440 North 13th St., Philadelphia.
Amy C. Bowman, 5129 Baltimore Ave., Philadelphia.
Mary Adele Evans, Wm. Penn High School, 15th and Mt. Vernon Sts., Philadelphia.
Elizabeth N. Frorer, Kensington High School, Amber and Cumberland Sts., Philadelphia.
Anna Kleefeld, 600 N. 34th St., Philadelphia.
Library, Temple University, Broad and Berks Sts., Philadelphia.
Elizabeth Linton, 3810 Locust St., Philadelphia.
Mrs. Margaret L. Moore, 402 W. Durham Rd., Philadelphia.
Edith E. Morin, 4816 Windsor Ave., Philadelphia.
Ida C. Wild, 1137 E. Tioga St., Philadelphia.
Emma Wolfenden, 5901 Ridge Ave., Roxborough, Philadelphia.
Frank C. Baldwin, 6401 North 7th St., Oak Lane, Philadelphia.
Prof. Frank Ehrenfeld, 214 N. Center, Phillipsburg, Pa.
Florence Holderman, 232 Second Ave., Phoenixville.
Miss Anna R. Graff, Latimer Junior High School, N. S. Pittsburgh.
Margaret Cosgrove, 48 King Edward Apts., Pittsburgh.
Lila C. MacMillan, 4626 Forbes St., Pittsburgh.
Prof. K. D. Swartzel, 4360 Center Ave., Pittsburgh.
Library Dept., University of Pittsburgh, 310 State Hall, Pittsburgh.
Elizabeth B. Cowley, 913 Arch St., N. S., Pittsburgh.
Lillian M. Frasch, 901 Highview St., Pittsburgh.
Helen L. Jacobi, 1410 Key Ave., So. Hills P. O., Pittsburgh.
Sarah B. Houston, Langley High School, Pittsburgh.
Jane Matthews, The King Edward Apts., Metwood and Bayard Sts., E. E., Pittsburgh.
Mary Anne Porter, 107 Center Ave., Bellevue P. O., Pittsburgh.
C. A. Leopold, 7931 Susquehanna St., Pittsburgh.
Herbert L. Spencer, Latimer Junior High School, North and James St., Northside, Pittsburgh.
Tressa C. Yeager, Washington School Bldg., Mt. Lebanon, Pittsburgh.
George G. Sheppard, 772 King St., Pottstown, Pa.
Elsie M. Eidam, 116 Robeson St., Reading.
Anna Gehrleindaub, 537 N. Eighth St., Reading.
High School for Boys, Library, 8th & Washington Sts., Reading.
Carrie M. Rhodes, 542 McKnight St., Reading.
Louisa L. Knabb, 830 Walnut St., Reading.
Miss Lizzie E. Miller, 639 Walnut St., Reading.
Miss Annie M. Swartz, Saylor Apts., 9th and Penn Sts., Reading.
Florence E. Hollenbacher, 937 N. 11th St., Reading.
Marie Reagan, Northeast Junior High School, 13th and Marion Sts., Reading.
Effie Holstein, Richland.
David L. Hoffman, 111 Welcome St., Ridley Park.
Helen Fudge, Rosemont.
Alice M. Grebe, 525 Walnut St., Royersford.
Central High School Library, Washington Ave. and Vine St., Scranton.
Beatrice R. Richards, 735 Wheeler St., Scranton.
Miss Leviah Sherrick, 510 S. Chestnut St., Scottdale.
Irene Williamson, 52 S. Oakland Ave., Sharon.

Reading Room, State Normal School, Shippenburg.
Nora A. Kieffer, Cumberland Valley State Normal School, Shippensburg.
Prof. Chas. S. Davis, Steelton Free Library, Steelton.
Geo. L. McCracken, 208 Vassar Ave., Swarthmore.
Swarthmore College, Dept. of Education, Swarthmore.
Emma L. Carlson, 102 W. Spruce St., Titusville.
Robert Schultz, 904 Foss Ave., Upper Darby.
Mrs. Olive L. Donnelly, Washington Ave., Manoa Heights, Upper Darby.

(To be continued)